Comments on a Study of Drift in Cesium Frequency Standards

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Abstract

This letter comments on a recent study of the drift rate of 115 cesium standards. The conclusion here is that no aging effect is visible in the results of the study.

A recent study [1] reported on statistics of the measured frequency drift rate ("aging") of 115 Agilent 5071A cesium-beam frequency standards. For each clock, one-day frequency measurements were taken over 155 consecutive days, and the slope of the least-squares line was calculated. The set of 115 slopes exhibited a bell-shaped histogram, a mean of -3.6×10^{-18}/day, and a standard deviation of 1.3×10^{-16}/day. Notwithstanding the excellence of these results, the study expressed some concern about a possible root cause of aging in these clocks.

It occurred to this writer that the results might be explainable as an effect of the random noise that we already know about from sigma-tau plots. After all, a random process observed over a finite time will almost always have a nonzero overall measured slope, even if the process theoretically has no long-term slope at all. Since the noise properties of 5071A clocks are well documented, it is possible to specify a noise model and to calculate the statistics of slope measurements. Let us assume a model of white FM plus flicker FM:

$$S_y(f) = h_0 + \frac{h_{-1}}{f}, \quad \sigma_y^2(\tau) = \frac{h_0}{2\tau} + h_{-1} \ln 4.$$

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Several authors [2][3][4][5] have computed the variance of estimated drift rate $\hat{d}$ (overall slope of $y(t)$ vs t) for several drift estimators in the presence of power-law noises. If $\hat{d}$ is the slope of the least-squares line, then

$$\sigma^2\left(\hat{d}\right) = \frac{6h_0}{T^3} + \frac{9h_{-1}}{T^2},$$

where T is the total duration of the data. (This formula holds in the limit of a large number of samples.)

With noise levels taken from [6], the components of Allan deviation and drift-rate standard deviation are as follows:

Noise	$\sigma_y(\tau)$	$\sigma\left(\hat{d}\right)$
White FM	$2 \times 10^{-11}\tau^{-1/2}$	1.22×10^{-16}/day
Flicker FM	5×10^{-15}	0.82×10^{-16}/day

The RSS of the $\sigma\left(\hat{d}\right)$ values is 1.47×10^{-16}/day. Thus, the observed standard deviation of the 115 drift rates, 1.3×10^{-16}/day, can be accounted for by random noise alone. Moreover, the observed mean drift rate, -3.6×10^{-18}/day, is well within the estimated standard deviation of the mean, $1.3 \times 10^{-16}/\sqrt{115} = 1.2 \times 10^{-17}$.

This writer concludes that the results of [1] are compatible with the hypothesis that the true drift rate is zero; in fact, the lack of any visible systematic frequency drift in this set of clocks is impressive. The use of the term "aging" in [1] suggests a physical mechanism behind the observed linear frequency drifts. For these clocks, though, it appears that one need not seek any special root cause of linear drift beyond whatever causes the random fluctuations.

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References

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