A Method for Using a Time Interval Counter to Measure Frequency Stability

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Abstract—**An interval timer can be used in a single-mixer frequency-stability measurement system in place of an event timer. The dead-time problem is avoided with the aid of a reference pulse train and an algorithm for ambiguity resolution.**

I. Single-Mixer Method

IN THE beat-frequency or single-mixer method of frequency-stability measurement, two sources at frequencies f_0 and $f_0 + f_b$ are mixed down to a sinusoidal beat note at frequency f_b, which typically is around 1 Hz. This sine wave is passed through a zero-crossing detector to obtain a square wave at the same frequency. The relative time deviation or fractional frequency deviation of the two sources equals f_b/f_0 times that of the square wave or, more precisely, its stream of upcrossings, which are spaced approximately $1/f_b$ apart.

For the direct measurement of the two-sample (Allan) variance of a sequence of events such as the beat-note upcrossings, one must be able to capture adjacent periods, or, to put it more simply, one must be able to measure the epoch of each event with none skipped. In prior art [1], [2], this has been accomplished by event timers with resolution from 0.1 μs to 1 μs that can latch each event on the fly for subsequent computer processing.

II. The Picket Fence

The event timers used in the system mentioned previously were custom-built. On the other hand, commercial time *interval* counters with nanosecond resolution, IEEE-488 interfaces, and moderate cost are readily available. Of course, any such counter has dead time between measurements; unaided, it can measure at most every other period of a stream of events. If these devices are to be used in a frequency-stability measurement system, this limitation must be overcome, if possible without resorting to methods such as bias functions [3]–[5] for making model-dependent adjustments to the results. One method for capturing each period is to use two counters per beat-note channel in alternation, one counter measuring the even periods, the other measuring the odd periods [6]. The method to be explained here uses only one two-input counter per channel, but does require the use of one other component, a divider or pulse generator that provides a 10-pps (pulses per second) reference signal, called the *picket fence*, which constitutes a reference grid in time relative to which the epochs of the beat-note upcrossings are determined as shown below.

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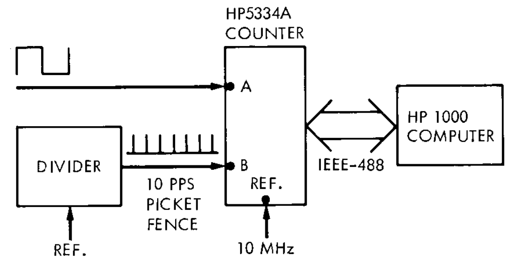

Fig. 1. Picket fence setup for measuring the stability of square wave.

The apparatus now used at Jet Propulsion Laboratory for processing a beat-note square wave of frequency below 2 Hz is shown in Fig. 1. The beat note and the picket fence signal go to inputs A and B of the counter. The same frequency standard drives both the counter and the divider to keep the picket fence coherent with the counter. In a multichannel system, each beat note has its own counter, and the same picket fence signal goes to all the B inputs.

To carry out a test, one first uses the "Period A" function of the counter to make a preliminary measurement of the period p of the beat note. Noise in this measurement does not influence the final results. Having measured the nominal period, one switches the counter to its "Time Interval A-B" function, and records all subsequent readings, starting on A and stopping on B. Each reading is the time interval between a upcrossing and the next picket fence pulse. Provided that the periods are not too short, the counter has time to reset itself between readings, and hence no upcrossing is missed. From these raw data, the actual upcrossing epochs are recovered in software by an algorithm given below. The time evolution of the measurement process is shown schematically in Fig. 2.

III. The Unfolding Algorithm

Let d be the picket fence period (100 ms), p the initial beat period measurement, and $v_0, v_1, v_2, \cdots$ the time

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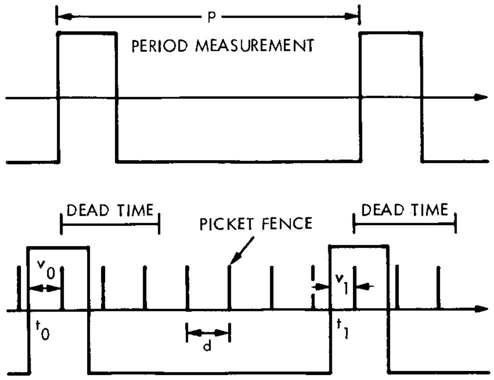

Fig. 2. Picket fence measurement process.

interval data corresponding to the unknown upcrossing epochs $t_0, t_1, t_2, \cdots$ relative to some arbitrary time origin (Fig. 2). Each t_n differs from the known quantity $u_n = d - v_n$ by an unknown integer multiple of d (notation: $t_n \cong u_n \pmod d$); we wish to resolve these ambiguities. The method for doing so depends on the regularity of the beat note that holds typically in an experimental situation, namely, that each period does not differ too much from some average of recent periods, although long-term changes may be great. To make this precise, let Δ denote the backward difference operator, e.g., $\Delta t_n = t_n - t_{n-1}$.

Assumptions

1) *Each period Δt_n is greater than $d + g$, where g is the maximum dead time of the counter:* This guarantees that no upcrossing is missed.

2) Define the sequence $p_0, p_1, p_2, \cdots$ by $p_0 = p$, $p_n = (1 - \lambda)p_{n-1} + \lambda \Delta t_n$ for some constant λ in [0, 1]. Thus, the p_n sequence is the output of a filter applied to the Δt_n sequence. *It is assumed that Δt_n differs from p_{n-1} by less than $d/2$.* The choice of λ is discussed below.

Since the t_n increase linearly without limit and may contain important information in their least significant digits, they are awkward to compute, store, and use. Accordingly, the algorithm given below actually computes the sequence of raw time residuals x_n defined by

$$x_n = t_n - t_0 - np, \qquad n = 0, 1, 2, \cdots, \tag{1}$$

which can be used directly in the computation of two-sample variance, for example. In order to recast assumption 2 in terms of the x_n, let $q_n = p_n - p$. Then the q_n satisfy

$$q_0 = 0, \qquad q_n = (1 - \lambda)q_{n-1} + \lambda \Delta x_n, \tag{2}$$

and assumption 2 is equivalent to the assertion

$$|\Delta x_n - q_{n-1}| < d/2. \tag{3}$$

An essential ingredient of the algorithm is the *signed residue* function S mod (x, d), which is defined to be equal to x minus the closest integer multiple of d to x. For example, S mod $(3, 5)$ = S mod $(-7, 5) = -2$. If x is halfway between two integer multiples of d, then it doesn't matter whether S mod (x, d) takes the value $d/2$ or $-d/2$. This function has two essential properties: 1) if $x \cong y \pmod d$ then S mod (x, d) = S mod (y, d); 2) if $|x| < d/2$ then S mod $(x, d) = x$. These properties imply the following simple result.

Lemma: Let A, B, and C satisfy

$$A \cong B \pmod d, \qquad |A - C| < d/2.$$

Then

$$A = C + \text{S mod}\,(B - C, d).$$

Proof: By properties (1) and (2), S mod $(B - C, d)$ = S mod $(A - C, d) = A - C$, as asserted.

The idea here is that C, which is close to A, is used to resolve the ambiguity between A and B. Recall now that $t_n \cong u_n \pmod d$. Hence, by (1), $\Delta x_n \cong \Delta u_n - p \pmod d$. With $A = \Delta x_n$, $B = \Delta u_n - p$, $C = q_{n-1}$, (3) and the lemma give

$$\Delta x_n = q_{n-1} + \text{S mod}\,(\Delta u_n - p - q_{n-1}, d). \tag{4}$$

An algorithm for the sequential computation of x_n can now be given.

Unfolding Algorithm

1) Measure v_0, let $q_0 = 0$, $x_0 = 0$.
2) For $n = 1, 2, \cdots$ perform the following steps:
3) Measure v_n.
4) Let z = S mod $(v_{n-1} - v_n - p - q_{n-1}, d)$,
5) $x_n = x_{n-1} + q_{n-1} + z$,
6) $q_n = q_{n-1} + \lambda z$.

Choice of λ*:* The only restriction on λ is $0 \leq \lambda \leq 1$. If λ is set to 0, then all the p_n equal p, all the q_n equal 0, and we are assuming that the beat periods always stay within $d/2$ (50 ms) of the initial period p. This does not allow the algorithm to follow large, slow changes in Δt_n. If λ is set to 1, then $p_n = \Delta t_n$, $q_n = \Delta x_n$, and we are assuming that each beat period is within $d/2$ of the previous period. This allows large, slow changes to be followed, but makes the algorithm vulnerable to large errors in the data: in a previous version of this paper [7], it was shown that a single error could cause subsequent period calculations to be off by d. It is suggested that λ be set to a small positive value such as 0.01. This gives q_n enough damping to protect against errors and missing data, and allows the algorithm to follow a constant drift rate $\Delta^2 t_n$ nearly as large as $\lambda d/2$ (depending on the amount of noise in the data). Criteria for optimizing λ have not been developed.

IV. Noise Floor Test

To measure the noise floor and test the integrity of the digital portion of the measurement system, a square wave of period $(10 - r)d$, where $r = (\sqrt{5} - 1)/2$, the Golden Ratio, was used in place of a beat note. This particular period was used instead of $10d = 1$ s in order to get a good mix of counter readings v_n [8, pp. 510–11, 543]. The same frequency reference was used for the counter, picket fence, and input signal. In a 108 600-s test, the time residuals (with the mean frequency removed) stayed within 6 ns peak-to-peak, with no tendency to accumulate as a random walk, and a two-sample deviation (relative to 1.066 Hz) of $1.3 \times 10^{-9}/\tau$ was observed. This shows that all the equipment maintained coherence at the nanosecond level and that the algorithm performed correctly.

V. Conclusion

We have shown that an interval counter, aided by a picket-fence reference signal and an algorithm for ambiguity resolution, can emulate an event timer in a single-mixer frequency-stability measurement system. Advantages of the technique are high precision, convenient interfacing, and low cost. A disadvantage is the vulnerability of the technique to missing data. In fact, if the beat period is sufficiently close to an integer multiple of d, then a missing upcrossing is invisible to the algorithm. On the other hand, if the offset between the sources under test can be adjusted so that the beat period differs significantly from an integer multiple of d, as in the noise floor test described above, then missing data will stand out clearly as large jumps in x_n, and it may be possible to repair the x_n after they have been collected.

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