Introduction to

Sapphire Microwave Frequency Sources

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Sapphire Microwave Frequency Sources

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Introduction: Why Sapphire?

- Ultra-Low Phase Noise
  - High Q values of 300,000 at room temperature, 30 million at 77K, and up to 10 billion at temperatures below 10K, reduce the effects of in-oscillator noise.
  - The high frequency of the microwave sapphire resonator additionally reduces the effects of white thermal noise.
  - No evidence so far of resonator flicker frequency noise to limit performance as there is for Quartz.
  - Low noise circuitry and components have been developed that take advantage of this high Q.
- Ultra-High Stability
  - Highest stability sources operating today for less than τ = 100 seconds measuring time use Helium-temperature sapphire resonators.
  - Thermal compensation at Ultra-Low temperatures by paramagnetic impurities
  - Promising preliminary results show quartz BAW stability with compensated operation at 77K, other thermal techniques being developed.
- Super Performance at Practical Temperatures
  - Superconductors require T < 2K
  - HTS so far not giving high Q's

How Sapphire?

- With its moderately high dielectric constant of (only) $\varepsilon \approx 10$, a sapphire resonator with a high Q would be impractically large if built using techniques previously developed for DRO's or optical dielectric resonators. This problem has been overcome by new methodologies:

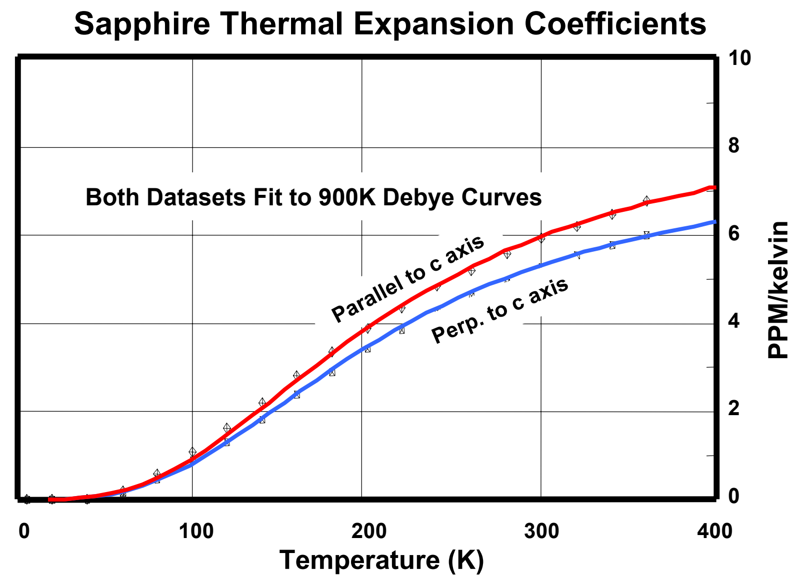
- The Whispering Gallery (WG) resonator is now the workhorse of this field. It allows the high Q inherent in the sapphire to be realized at any temperature
  - This mode is an analog of the phenomenon of Total Internal Reflection -- similar to bending light fiber back on itself to make a resonator
  - It necessarily has a number of waves around its perimeter -- total internal reflection only really works for plane waves.
  - WG resonators include a metallic or superconducting conducting (shielding) can: Resonator design involves trading off the significant rf losses at the can wall with overall size and ease of RF coupling.
- Sapphire/Superconductor resonators have been under study for some time at Helium Temperatures, and more recently, using HTS materials at 40K and up. This approach makes possible a small resonator with a low excitation number.

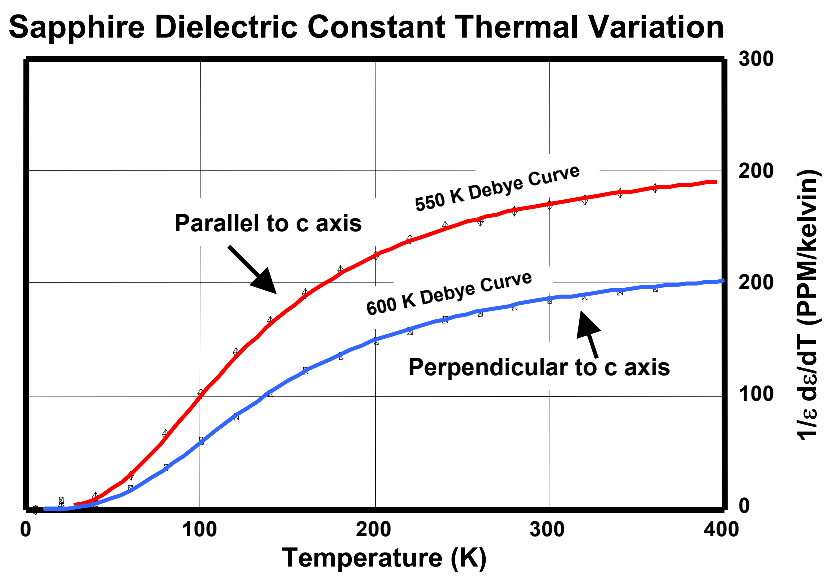
Sapphire Properties

- Physical:
  - Sapphire is the crystalline form of Al_2O_3, a hard and transparent mineral. It has a relatively high Debye Temperature of 1024 K for a low expansion coefficient. Its hexagonal crystal structure gives it a preferred c axis in which direction it exhibits different properties from those for the other two axes, which are identical. An inexpensive polycrystalline form known as Alumina is widely used as a microwave substrate. Sapphire can be doped with small quantities of metal ions other than Aluminum. For the case of added Chromium, the resulting mineral is known as Ruby.
- Electrical:
  - Sapphire is transparent in the visual part of the spectrum, becoming highly absorbing in the infrared beyond 6 μ. Microwave and RF losses are the lowest for any known solid. It has an index of refraction of n = 1.76 at optical frequencies and low-frequency dielectric constants of ε par. = 11.589 along the c axis and ε perp. = 9.395 for the other two directions (at room temperature.) This substantial anisotropy must be explicitly taken into account in order to design microwave resonators containing sapphire elements.

Sapphire Thermal Dependencies

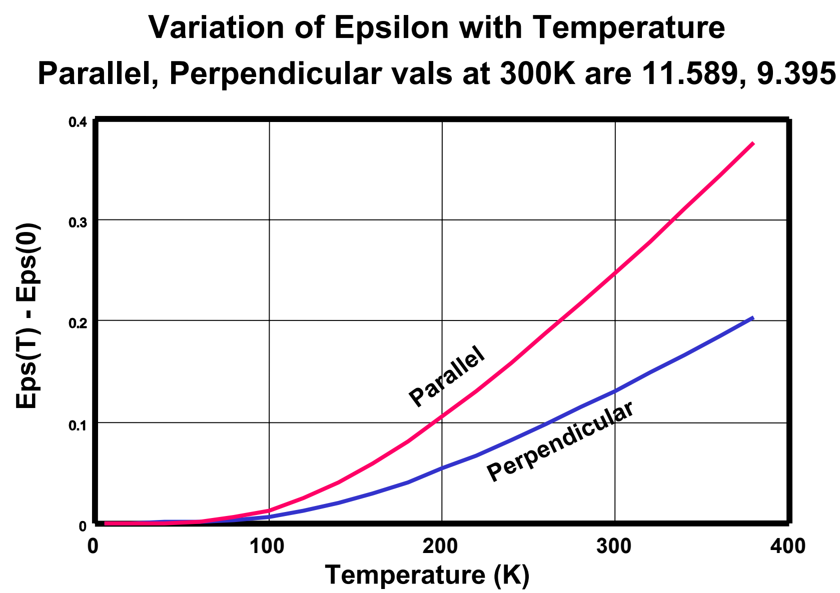
- The Expansion Coefficients at room temperature have values of about 6 PPM/K, reducing to the T^3 dependence shown by all solids at low temperatures. Thermal dependencies are well fit by the calculated Debye energy for a Debye temperature of 900 K
- Dielectric Constants also show a Debye dependence on temperature, but with a greater anisotropy and with substantially larger values than for the expansion coefficients. Their variation with temperature is also greater than that of thermal expansion, with fitted Debye temperatures in poor agreement with heat capacity values of $T_D \approx$ 1130K. Agreement of these low-frequency measurements with the Debye form however, is excellent.

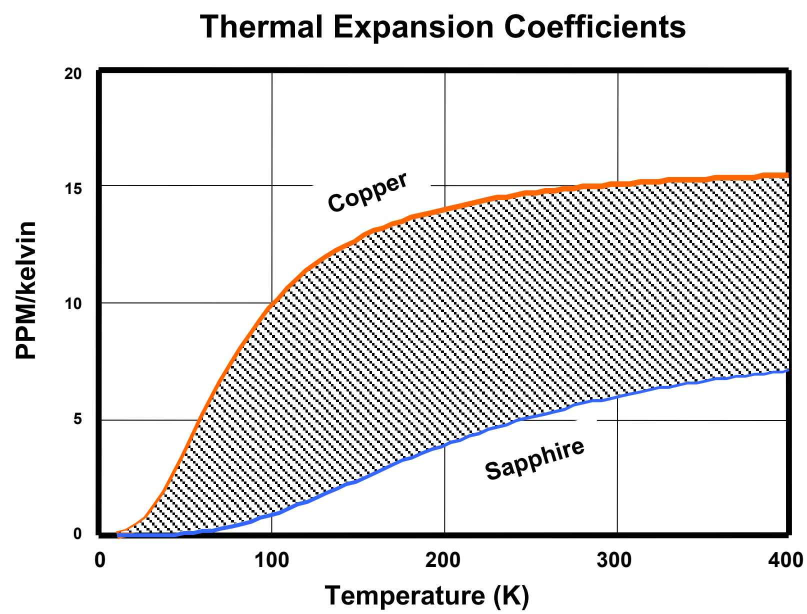




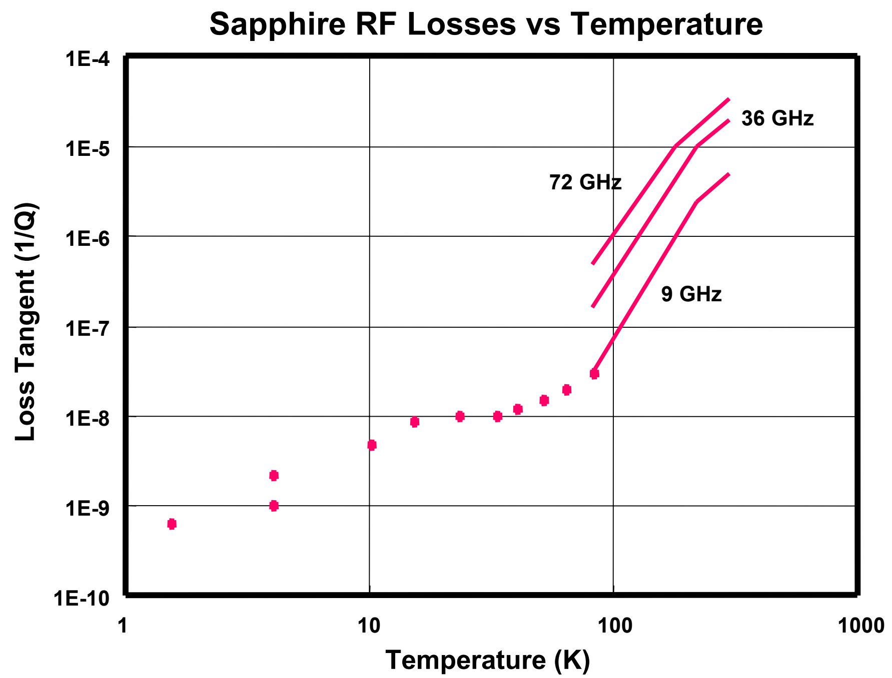
Other Thermal Aspects

- Actual values of the dielectric constants are shown at the right. These curves can be obtained by integrating the coefficients shown on the previous page. While the dielectric constant parallel to the **c** axis is only 23%, larger than that in the perpendicular direction, the temperature variation is nearly twice as large.
- Most metals have lower Debye temperatures than sapphire, resulting in higher expansion coefficients, especially at low temperatures. The graph on the right shows a copper expansion coefficient 2x larger than sapphire at 300K, increasing to nearly 10x larger at 77K. The copper data shows a good fit to calculations for a 330 K Debye temperature.





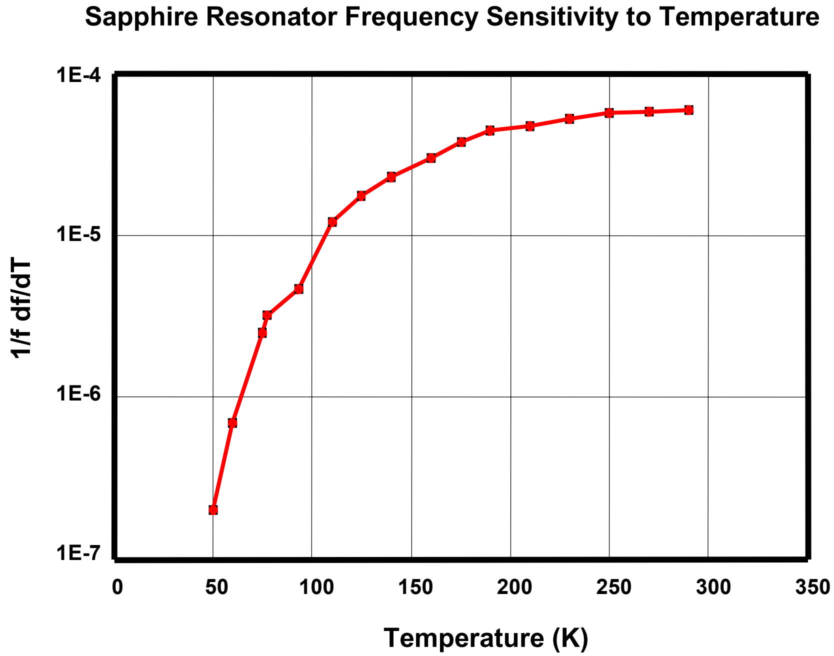
The High Sapphire Q



- Sapphire dielectric loss is so low that much published data is wrong, even at room temperature.
  - Losses are much lower than metallic losses for low order modes.
  - Whispering Gallery Q measurements allow inherent losses to be measured for the first time by eliminating metallic losses. Comparison is $Q = 1/\delta$ where δ is the loss tangent.
- The Dielectric loss shows an approximate T^5 dependence on operating temperature.
  - Very rapid Q improvement as temperature is reduced.
  - No "Transition Temperature" as characterizes High Temperature Superconducting (HTS) Resonators.
- Frequency dependence approximates $\delta \propto \omega$ where ω is the microwave frequency.
- Copper Q's are typically 10,000 - 30,000 at Microwave frequencies

Temperature Dependence of Resonator Frequency

- Frequency varies strongly with temperature - primarily due to variation of dielectric constants. (Blair and Evans)
- Excellent Phase performance still possible at high temperatures
  - Problem: Room temperature Q of $3x10^5$ together with $6x10^{-5}$/kelvin frequency coefficient $\rightarrow$ 1/18 kelvin variation moves 1 bandwidth.
  - Use slow temperature lock loop to keep frequency "on",



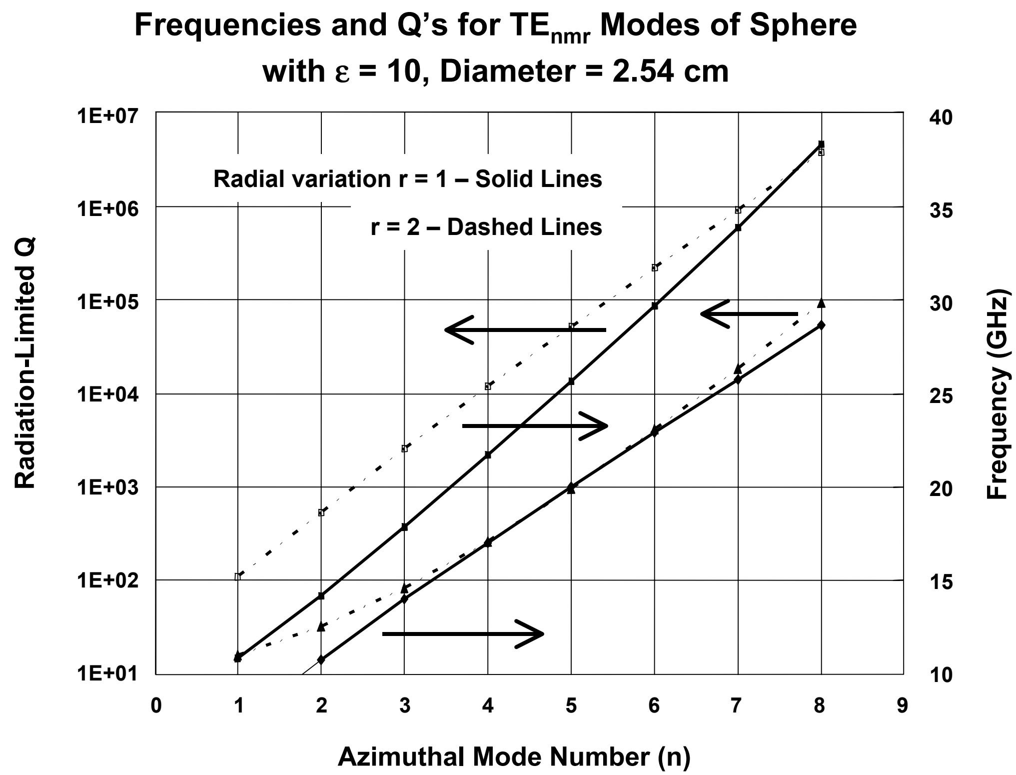
- Resonator will not provide high stability with uncompensated operation at temperatures of 40 K and above.
- Helium-temperature systems now show parts in 10^{16} stability.
  - Q's of 10^9 together with paramagnetic ion compensation for highest stability. Whispering-Gallery systems give X-band compensated operation at 6K
  - Fundamental TE_{011} Resonators at S-band with superconducting coatings give compensated operation at 1.5K

Temperature Dependence of Resonator Frequency (cont'd.)

- Size of sapphire increases as temperature is raised .
  - $\rightarrow$ Frequency decreases about 6PPM/kelvin at room temperature.
- Dielectric constant of sapphire increases as temperature is raised
  - $\rightarrow$ Frequency decreases 45 - 65 PPM/kelvin at room temperature depending on field orientation.
  - Frequency variation is just half the dielectric variation since $\omega \propto (LC)^{-1/2}$
- Copper container size increases as temperature is raised
  - $\rightarrow$ Frequency decreases 15PPM/kelvin * wall field reduction factor (if any).
- Detailed comparison shows good agreement of Whispering Gallery mode frequency variation shown at right with low-frequency dielectric constant thermal dependency shown earlier.

Whispering Gallery Mode Resonators

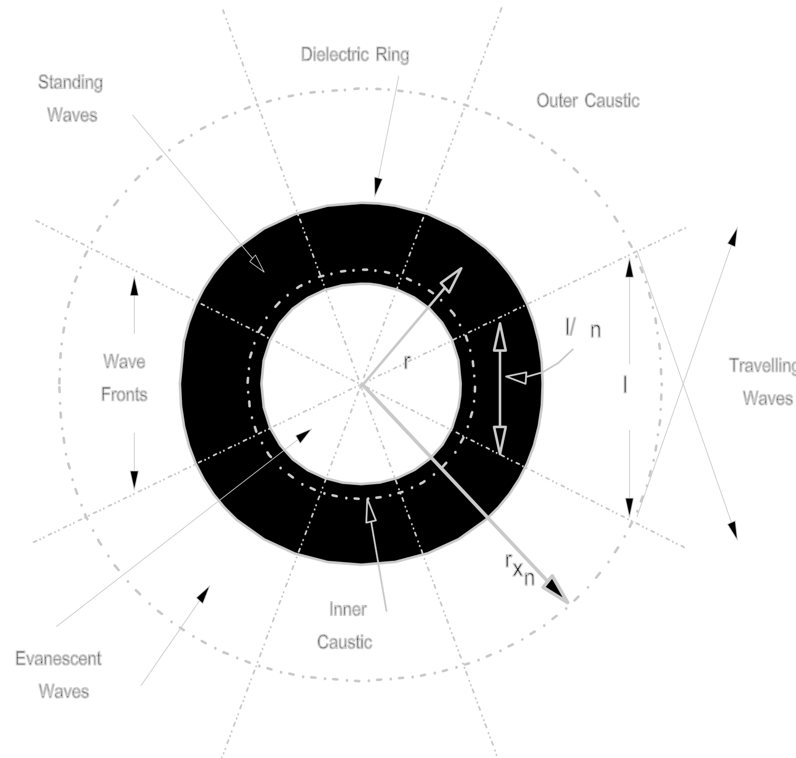
- It is instructive to look at the modes of an isotropic sphere.
  - Analytical form available -- numerical solutions can be easily generated.
  - High radiation-limited Q's are possible.
- While Q values for isolated sphere are high, they require a large size to match cryogenic sapphire Q values.



- For coupling and isolation a container is desirable -- and allows a significantly smaller package.

Whispering Gallery Modes (continued)

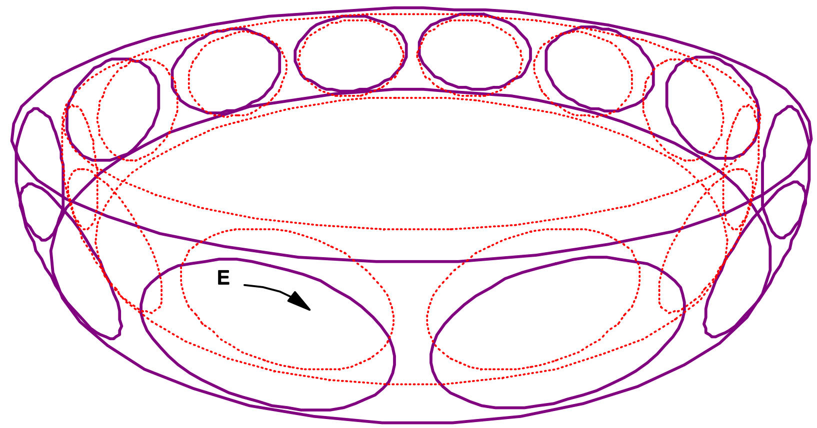
- Four Regions Describe radial Field Configuration
  - Inner Evanescent Region (inside inner Caustic)
  - Standing Wave Region
  - Outer Evanescent Region
  - Traveling Wave Region
- Evanescent Regions show exponentially decaying fields.
  - Result in small field values outside sapphire and also on the cylindrical axis.



  - Is a consequence of the wave equation -- $k_\varphi^2 + k_r^2 + k_z^2 = \varepsilon\, \omega^2 / c^2$. Thus the large values for k_φ due to waves propagating azimuthally in the dielectric require k_r be imaginary just outside the sapphire, until the outer caustic is reached, where k_φ is reduced to its free-space value.
- Proper Placement of Can is Far Enough out to give good isolation, but no so far as the Outer Caustic.

Whispering Gallery Modes (continued)

- The Field Configuration for the fundamental (lowest frequency) mode depends on its geometry.
  - It can be either WGE_{n11} with Electric fields forming loops in the plane of a thin sapphire disk,
  - or WGH_{n11} with Electric field loops in the **φ-z** plane for a taller wheel geometry shown on the right..

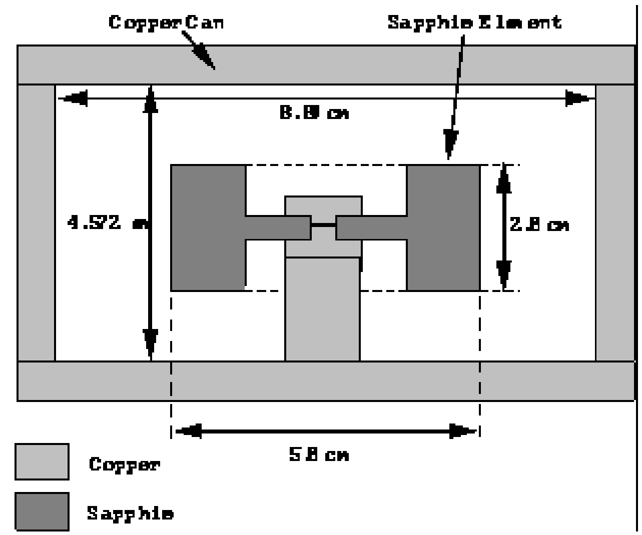


Electric Field Configuration for WGH_{611} Mode

- Because of the anisotropy of the sapphire dielectric constant, the WGH modes experience a slightly higher dielectric constant than the WGE, which have their electric energy almost entirely in the **r- φ** plane which is *perpendicular* to the **c** axis. The WGH modes, on the other hand, have their energy more or less evenly divided between *parallel* and (larger) *perpendicular* dielectric constants, the exact balance depending on the ellipticity of the E-field loops. (High values for **n** show a vertical ellipticity with more energy *parallel* to the **c** axis and a higher effective dielectric constant.

Whispering Gallery Modes (continued)

- While the high dielectric constant of sapphire tends to confine the E fields to the sapphire element, the H fields extend beyond the sapphire itself-- primarily in the **z** direction for WGE modes and in the **radial** direction for the WGH modes. These fields will tend to cause losses for metallic surfaces of the containing can in this principal direction. Because container diameter is often more difficult to achieve than length, this gives an advantage to the WGE mode family.



Wheel-type resonator cross-section

- Modes with greater number of variations in the **r** or **z** direction, e.g. WGH_{n12} with two circles of E (one above the other) will generally have higher frequencies, and so the evanescent regions are less effective in isolating microwave energy to the sapphire element itself (invoking the wave equation again.)
- Each of the modes discussed so far is actually doubly degenerate, with a second mode shifted in phase by 90 degrees from the mode shown. Mode splitting due to sapphire imperfections tends to be very small (parts per million) and with random orientation. Thus some means must usually be found to split the modes and control the mode orientation. An exception to this case is for the lower Q room-temperature resonators where the stronger input coupling dominates the small mode splitting.

RF Coupling

- Loop coupling has a long history for use with high-Q cryogenic resonators
  - Variable coupling is easily achieved by use of moveable loop
  - Same design can be used for modes with orthogonal field patterns—rotate 90 deg.
  - Wide range of coupling strengths are possible in WG resonators due to exponentially increasing field strength away from the wall.
  - Withdrawing loop into tubular access hole allows very weak coupling as required by superconducting resonators
- Waveguide coupling is a good match for developed designs
  - Coupling strength typically turns out to be somewhat overcoupled for WG resonator designs with copper walls, 10% wall losses.
    - Solution is to use next smaller waveguide size, with a short section below cutoff
    - Post-construction variability provided by variable length Teflon insert.
  - Orientation and mode selection are very well defined
- Coupling elements also break the degeneracy of the two orthogonal WG modes allowed by cylindrical symmetry.
  - Mode splittings of typically 10^{-6} due to mechanical imperfections are dominated by waveguide couplers at 300K where Q's are typically 250,000
    - This gives one strongly coupled and one weakly coupled mode as desired
  - Contrarywise, for ultra-high stability systems with $Q \geq 10^9$ waveguide couplers have no apparent mode-splitting effect
    - Result is random mode orientation for each sapphire resonator part—sometimes must adjust orientation to give appropriate coupling strength to desired mode.

Turnover Temperature—Paramagnetic Spin Tuning

- Competing paramagnetic spin (1/T) and Debye expansion (T^4) frequency variations can provide compensation in sapphire and ruby
  - Sapphire with incidental chromium spins can show compensation below zero field splitting which is 11.44 GHz
  - Newly available sapphire without chromium (careful separation of sapphire and ruby processes by manufacturer) shows frequency independent compensation in microwave range since splittings of Ta, Mo, impurities are 100 - 1000 GHz.
  - Microwave coupling to spins depends on mode
    - WGH modes couple strongly to spins, WGE modes don't
- Practice of using incidental levels of paramagnetic impurity concentrations is inappropriate for anything but one-off demonstrations.
  - Turnover temperatures typically 4K - 6K for WGH modes for both old and new sapphire processes
    - Temperatures show too much variation for cryocooler operation
    - Temperatures are too low for vibration-free and reliable cryocooler cooled Dewar design
    - Of two new sapphire process resonators previously tested, both showed good Q, but one had turnover of 14K which is unusable
  - Achieving 2 crucial and technically challenging parameters (T_T and Q) in every sapphire is scary
  - At mercy of manufacturer's processes and markets

Turnover Temperature (cont'd.)

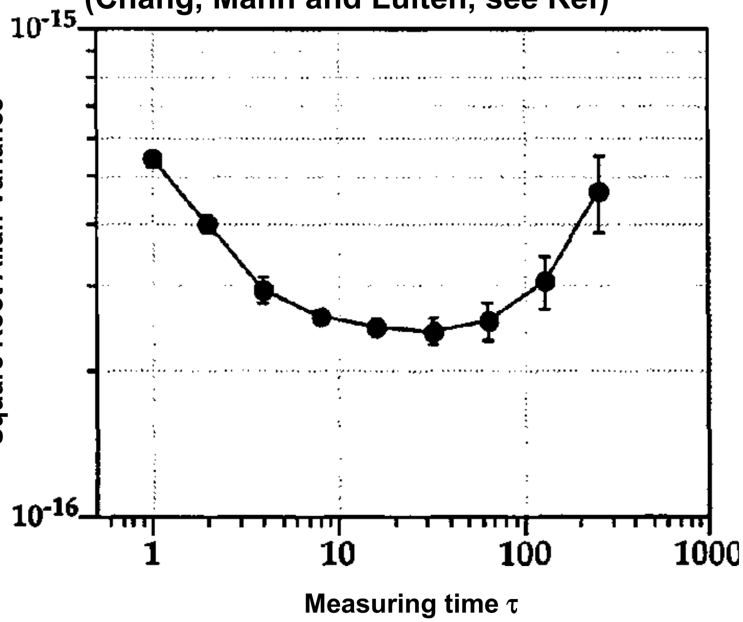
- Low Temperature Compensated Resonators (continued)
  - With an operating frequency below the zero field splitting for the paramagnetic spins involved, their rf susceptibility shows a 1/T temperature dependence, with a sign that tends to compensate sapphire's natural variation
  - Incidental impurities in as-made samples of high purity sapphire typically give turn-over temperatures between 4K and 14K. Compensation is typically strongest for WGH modes, depending on the impurity and on the resonator configuration. Sometimes no turnover is seen.
  - Impurities can be added in the manufacture of the sapphire part to gain more control over the compensation process.
  - An additional resonator element may be added with a high concentration of paramagnetic impurities (e.g. Ruby).
- Example technologies:
  - Helium-cooled oscillators built at Univ. of Western Australia (UWA) using superconducting niobium containing cans that have demonstrated frequency stability in the 10^{-16} range with operating temperatures below 6K
  - Cryocooled oscillators built at JPL for installation in Nasa's Deep Space Network in support of the CASSINI Ka-Band Experiment that show stability of a few parts in 10^{15} together with year-long operation.

Internal Compensation

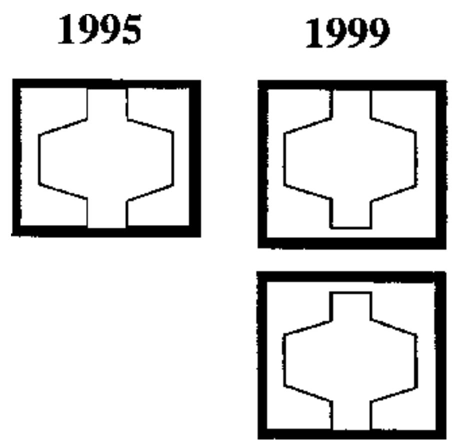
UWA Helium Temperature Sapphire Oscillator

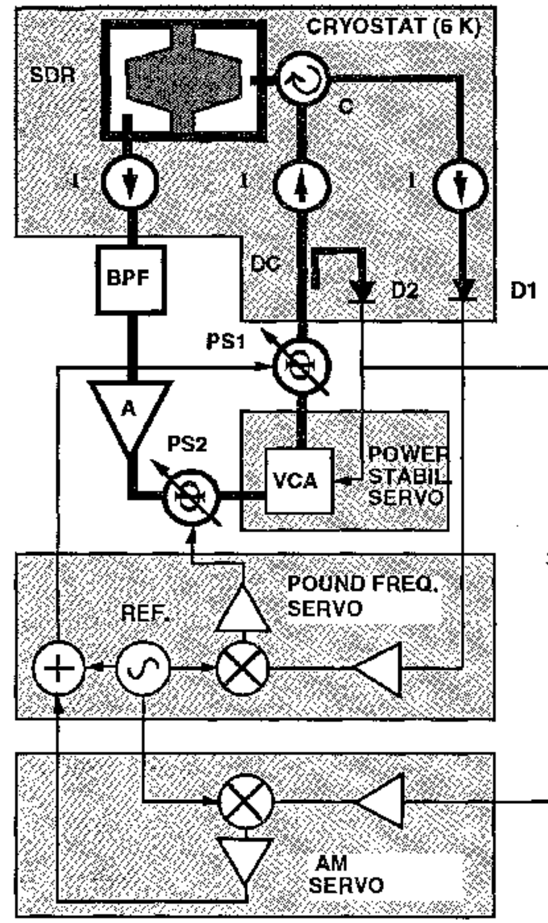
- Recent results shown in the graph below show the truly outstanding frequency stability that can be obtained with this technology
- 30 years ago, Stein and Turneaure showed similar results with superconducting cavities at ~1K, but none since till now
- Compensation is by incidental paramagnetic impurities at temperature of about 5K, Q's are 1-5x10^9
- Advanced electronics include rf power regulation, interferometric techniques

Frequency Stability for a single Sapphire Oscillator
(Chang, Mann and Luiten, see Ref)



New resonator is supported at only one end





Loop Oscillator Circuit

Internal Compensation

JPL Superconducting Cavity Maser Oscillator

- Helium-cooled active oscillator showed stability of about 1×10^{-15}, operating at ~1.5K compensated by incidental paramagnetic impurities.
- Three-cavity design selected operation in high-Q mode.
- RF pump frequency of 14GHz provides power
- About 10^{-9} W output power

Superconductor on Sapphire High-Q Resonator

Ruby active element in ¼ wave coaxial resonator

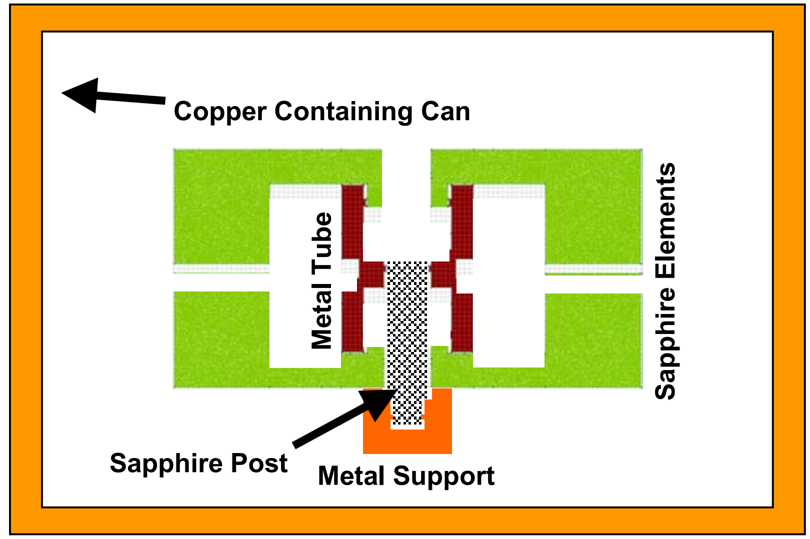
External Element Compensated Resonator Design

- Stable frequency requires operation at resonator turn-over temperature
  - At 8K, uncompensated frequency slope is $3x10^{-9}$/K, would require 300 nano-kelvin regulation to achieve 10^{-15} stability
- Basic Idea -- Externally compensated resonator as demonstrated in 77K CSO
  - Add external element with opposite coefficient of frequency variation with temperature
    - Thermal design of resonator is crucial to performance -- if the two elements aren't at the same temperature it won't work
    - Need much weaker compensating mechanism at 10K than at 77K due to T^4 Debye dependence of frequency on temperature
    - Compensating mechanism must not destroy resonator Q
  - Develop design with thermally attached but weakly coupled ruby element
    - Sapphire resonators are already somewhat compensated (WGH modes) due to incidental paramagnetic impurities with 1/T frequency dependence
      - Naturally occurring paramagnetic impurities do not apparently ruin the Q so could possibly strengthen the compensation and raise the turn-over temperature from natural 4-6K to 8-10K
      - Early results on ruby indicate that Q's are probably still ok at fairly high Cr concentrations
    - Need to determine for sure ruby is ok
  - Design must not increase acceleration sensitivity -- would increase effects of cryocooler vibrations

External Element Compensation (cont'd.)

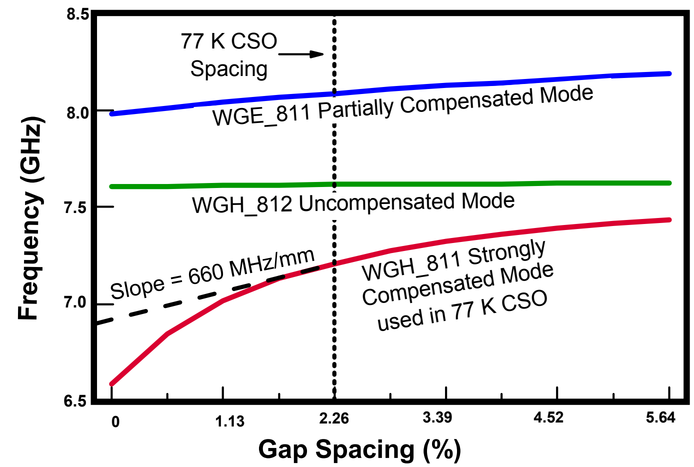
Thermomechanical Expansion

- A variety of methodologies have been proposed, investigated, and demonstrated to give higher turnover temperatures without to great a loss in performance. These Mid-Temperature Compensated Resonators (30K < T < 100K) should make possible stability of 10^{-14} or better, while capable of being cooled by inexpensive and small single-stage cryocoolers. These include:
  - Resonators with added dielectric elements with inverse temperature dependences from that of sapphire. The most promising of these is Rutile, and sapphire resonators compensated at temperatures of 40-60K have already been demonstrated.
  - Thermomechanically compensated resonators. Several examples have already been demonstrated with stabilities of 10^{-12} to better than 10^{-13} in the 80K temperature range. A new capability has been proposed with an operating temperature of ~35K that promises a stability of 10^{-14}.
  - Resonators with Bragg-type configurations that promise high Q's by keeping electromagnetic energy largely outside the sapphire itself.



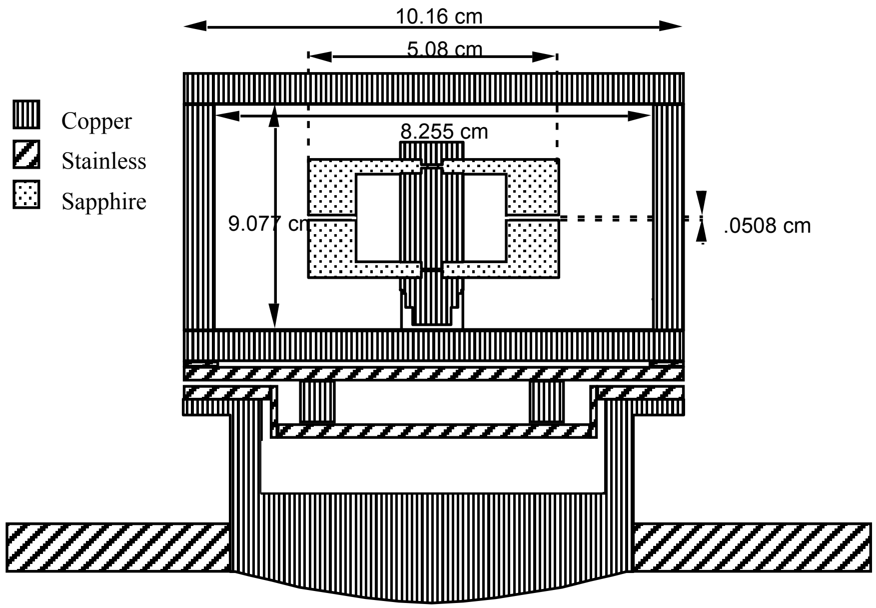
Thermomechanically compensated resonator

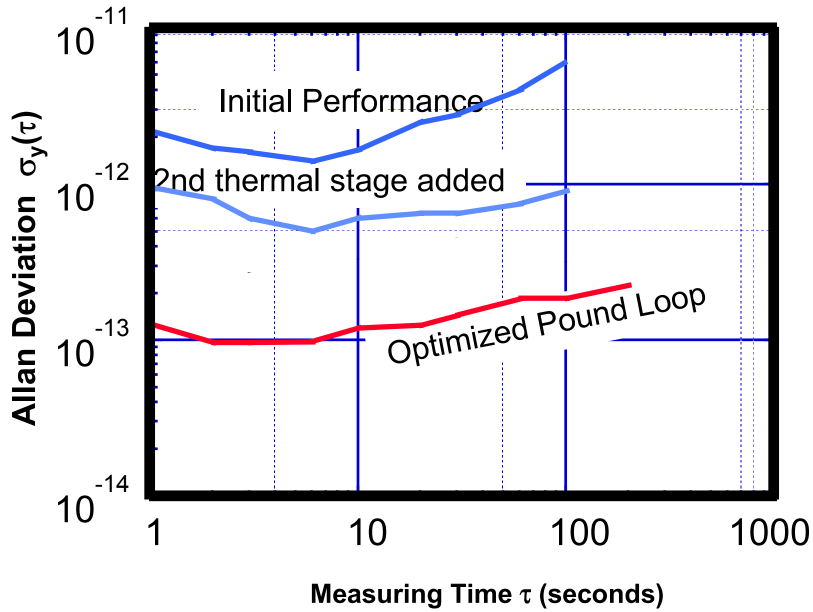
FEM Calculation of Frequency Variation with Gap for Several Mode Families



External Element Compensation (cont'd.)
Thermomechanical Expansion

- 77K CSO developed at JPL showed stability of 1.0×10^{-13}
- Advanced Pound techniques gave best stability to date for sapphire oscillator operating above ~10K even with resonator Q of only 2×10^{6}
- High stability required several stages of thermal isolation
- Problem is high drift rate

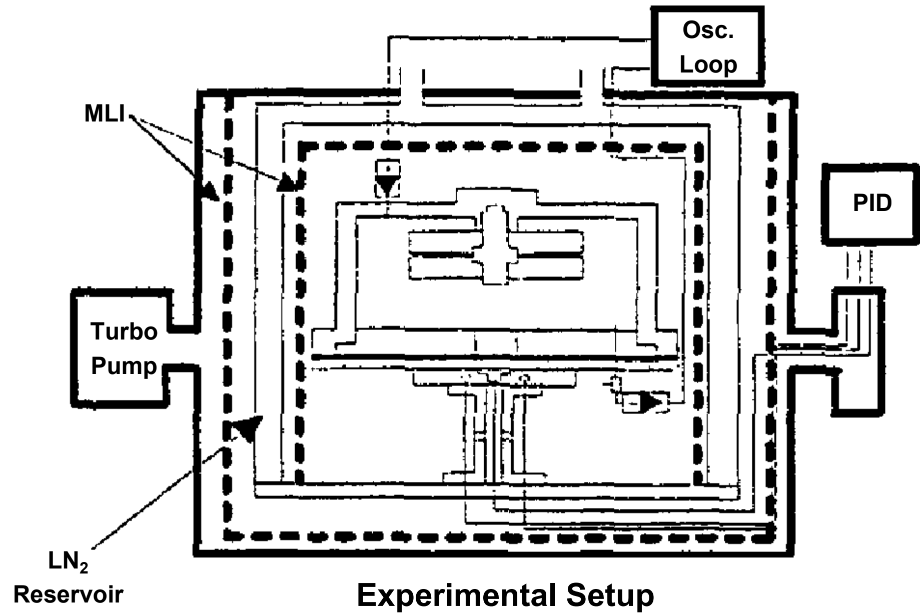




External Element Compensation (cont'd.)

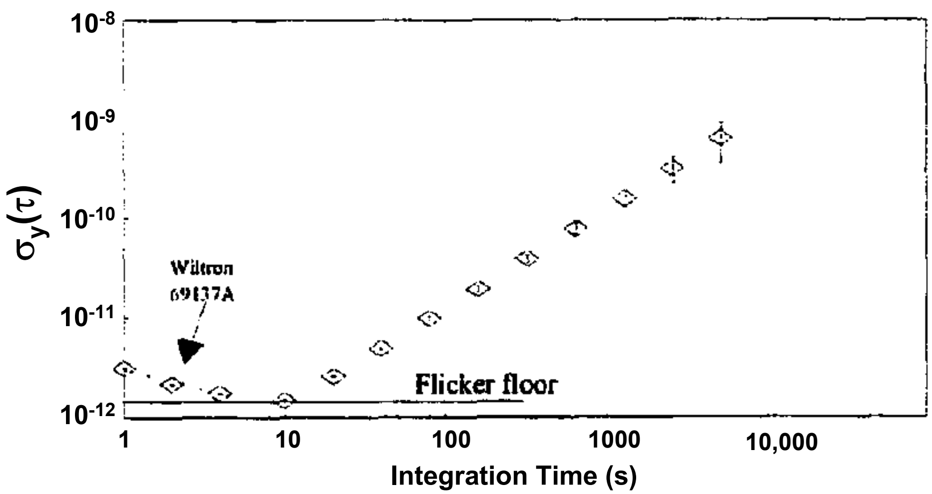
Thermomechanical Expansion

- System showed high stability of approximately 10^{-12}
- Temperature turnover ~88K for easy operation with LN^2 cooling

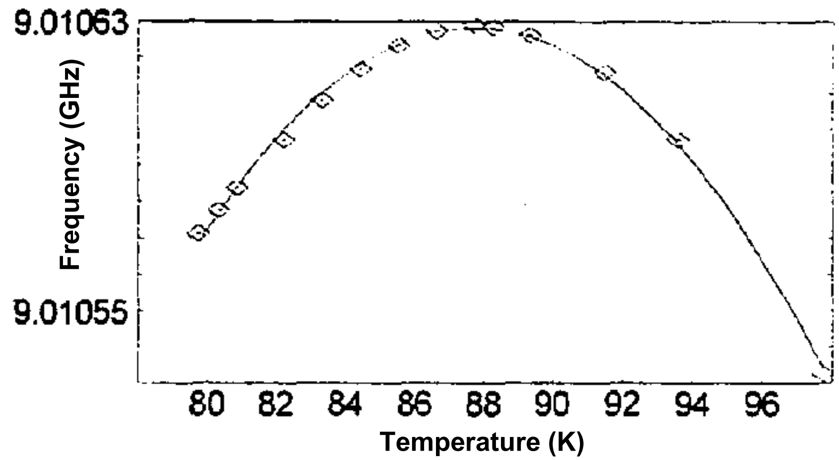


Experimental Setup

Frequency Stability (Kersale, Giordano, Lardet-Vieudrin, Lajoie and Chaubet, see Ref)



Frequency Variation with Temperature



External Element Compensation (cont'd.)
Dielectric Material (Rutile)

- Rutile is shown to be most advantageous of various available dielectrics with appropriate dielectric variation with temperature
  - Add material with opposite coefficient of frequency variation with temperature as sapphire
  - Require combination of low dielectric loss together with rapid change of dielectric with frequency
- Study by Tobar, Krupka, Hartnett, Geyer and Ivanov (see Refs. and Figs. at right) showed advantage for Rutile over strontium lanthanum aluminate and YAG
- Disadvantage is a very high dielectric constant—a problem in resonator design

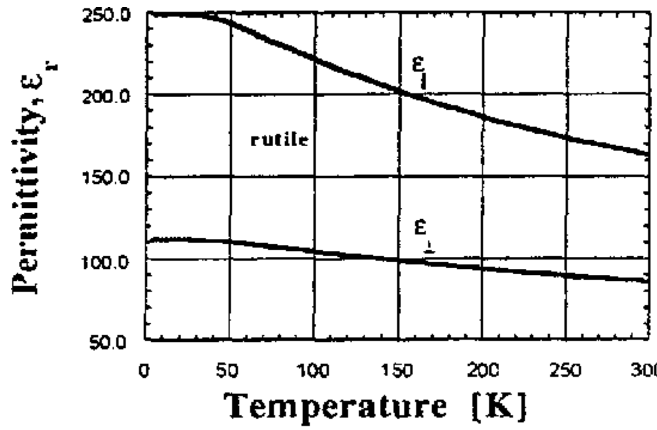


Fig.4. Permittivity versus temperature for rutile.

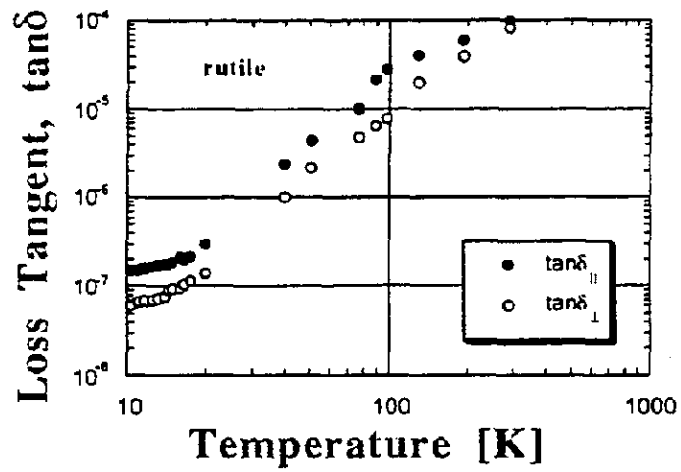
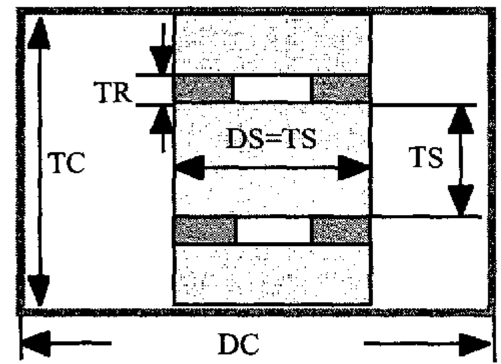


Fig. 9. Loss tangent versus temperature for rutile.

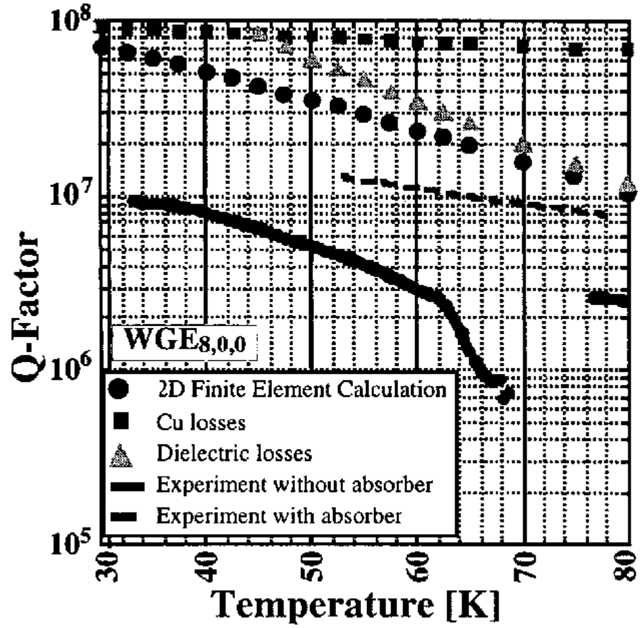
External Element Compensation (cont'd.)

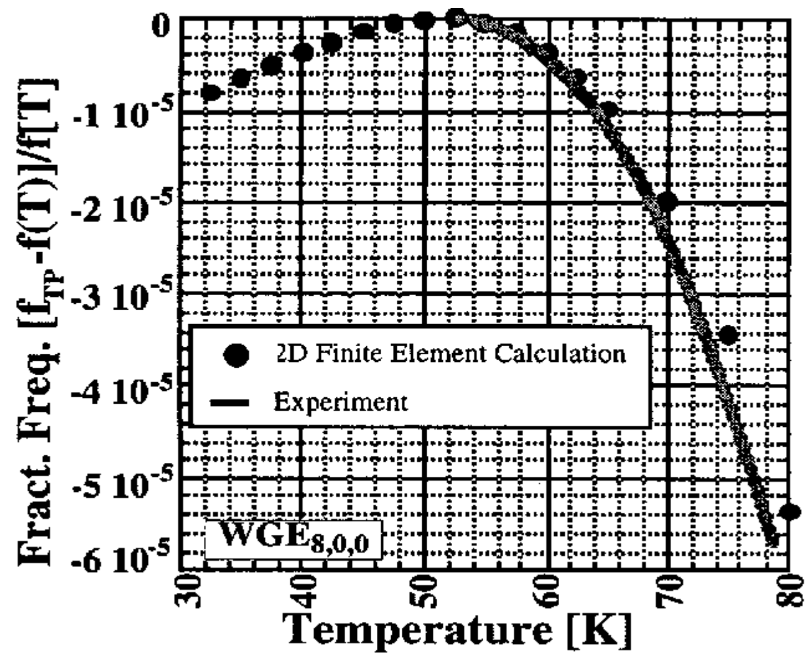
Dielectric Material (Rutile)

- Study by Tobar, Hartnett, Duchiron, Cros, Ivanov, Blondy and Guillon demonstrates advantageous turnover of ~50K together with Q's of ~10^7. No stability results yet.
- Also demonstrated novel resonator configuration with Bragg reflections to reduce energy density in dielectric



Sapphire-rutile DBRR with two layers of rutile and sapphire.

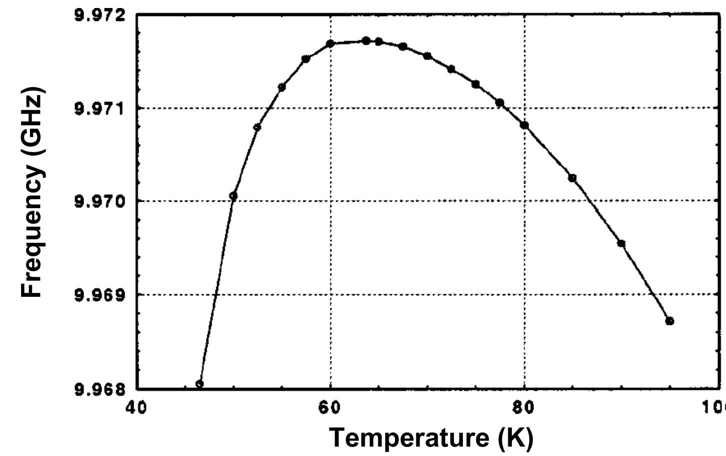


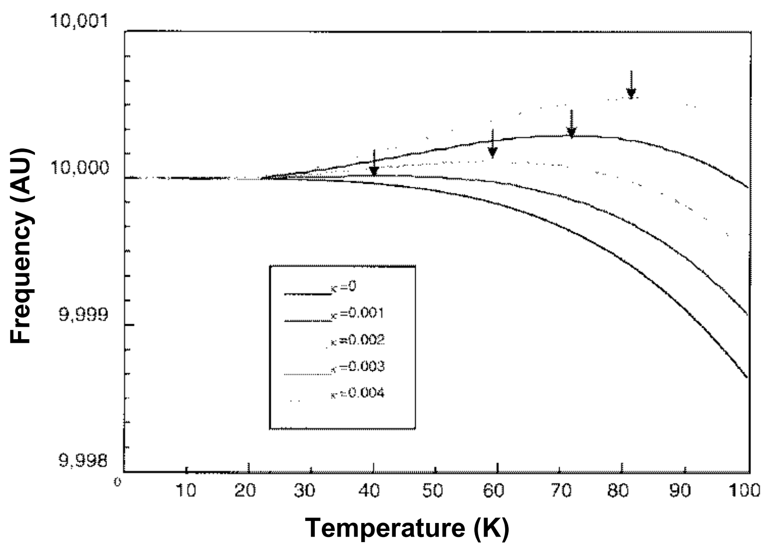


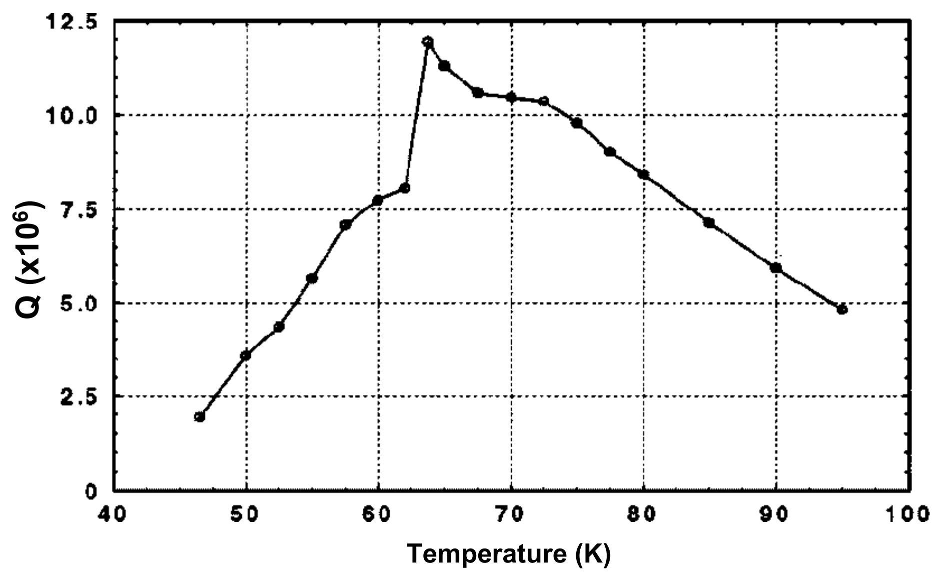
External Element Compensation (cont'd.)

Dielectric Material (Rutile)

- Oscillator built and tested by Winter, Klein, Hao and Gallop (see Refs.)
- Cryocooled oscillator shows turnover temperature ~60K together with Q of $\sim 10^7$.

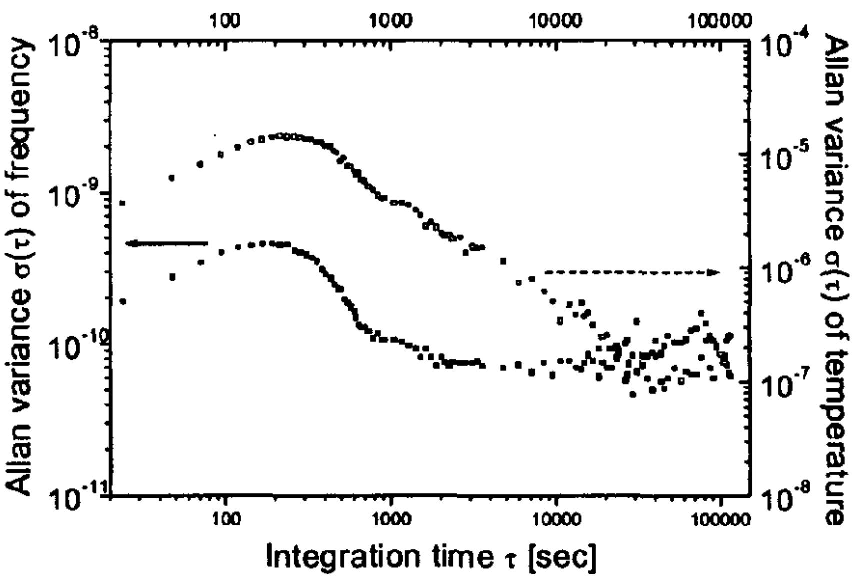


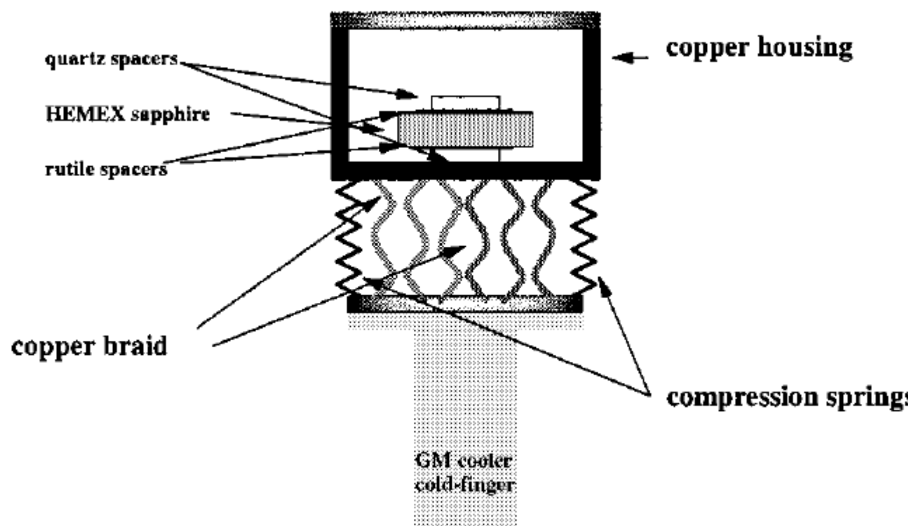




External Element Compensation (cont'd.)
Dielectric Material (Rutile)

- Winter, Klein, Hao and Gallop (cont'd.)
  - Frequency stability found to be closely related to temperature stability
  - A Rutile-sapphire time constant of several hundreds of seconds could explain the variability



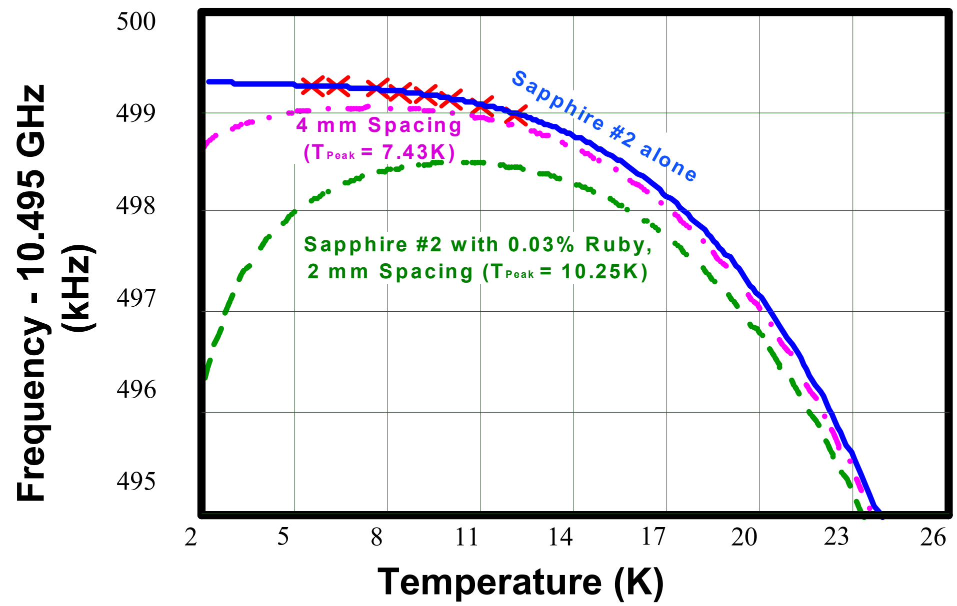


External Element Compensation (cont'd.)

Paramagnetic Spins (Ruby)

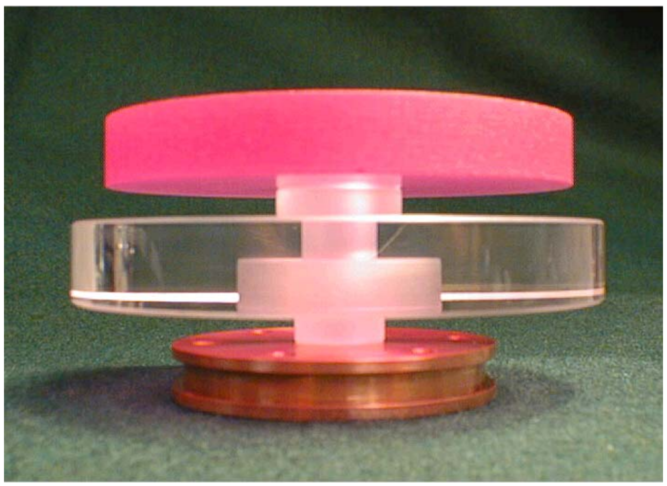
- Completed detailed compensation design
  - Measured ruby turnover temperatures are 20K - 40K, depending on mode, sapphire may or may not turn over
  - Thermal characteristics for both ruby and sapphire to be well modeled by 1/T and T^4 terms corresponding to Paramagnetic and Debye terms
  - Predict "as is" ruby turnovers of 7.43 K or 10.25K for 2mm and 4mm spacings, allows optimized operation without modifying the ruby elements

Sapphire, Expected CSO Temperature Dependence

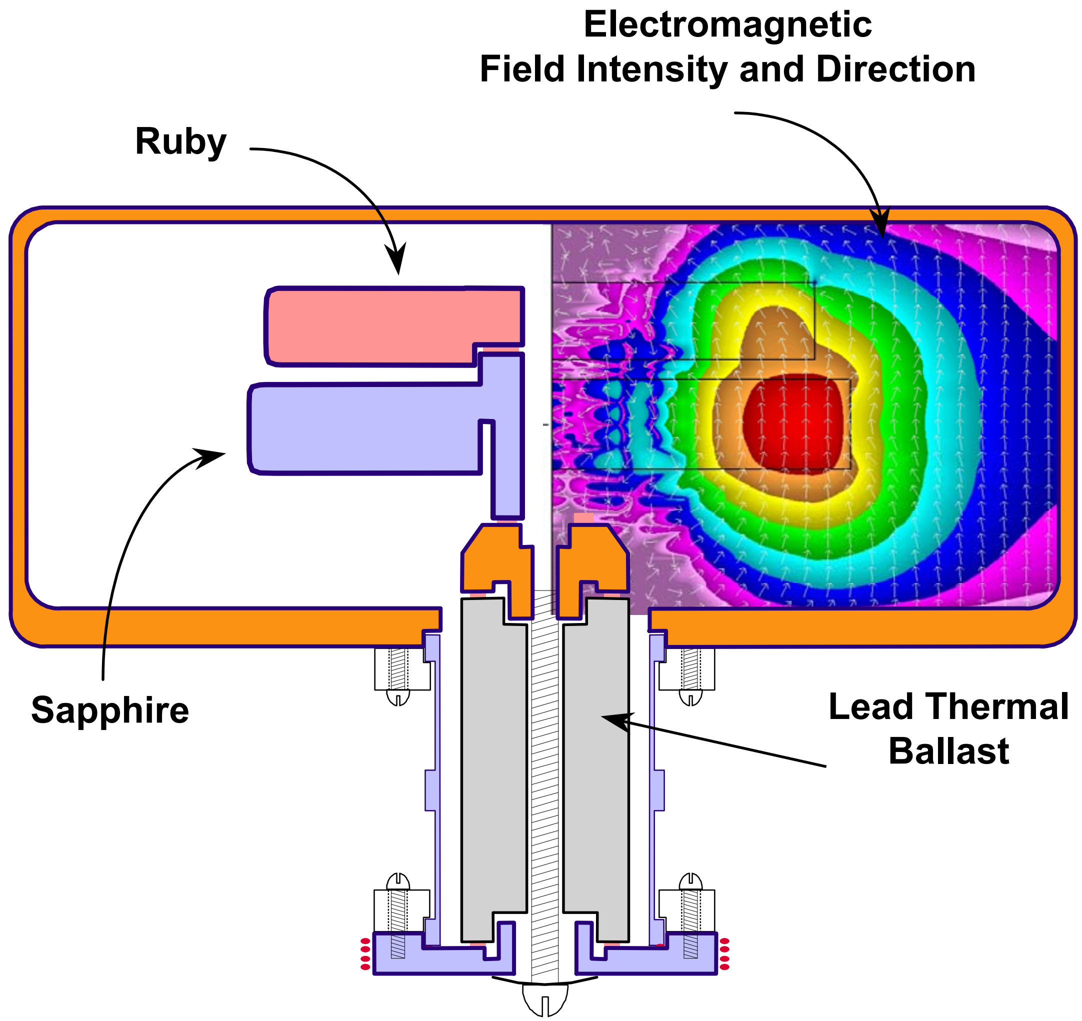


- Developed procedure for resonator evaluation
  - Adjust finite element model to show exact WGE mode frequency for Sapphire to match isolated sapphire resonator
  - Adjust 2 parameters of FE model for exact agreement with both WGE and WGH modes of isolated ruby resonator
  - Make FE calculation of magnetic EM energy in ruby sample when sapphire mode excited
  - Estimate energy in ruby WGM mode from B field angles in field views
  - Combine ruby 1/T component at 10.4 GHz with measured T^4 component for sapphire sample to predict turn-over temperature

10K Compensated Resonator Design and Components



Ruby and Sapphire Resonator Elements



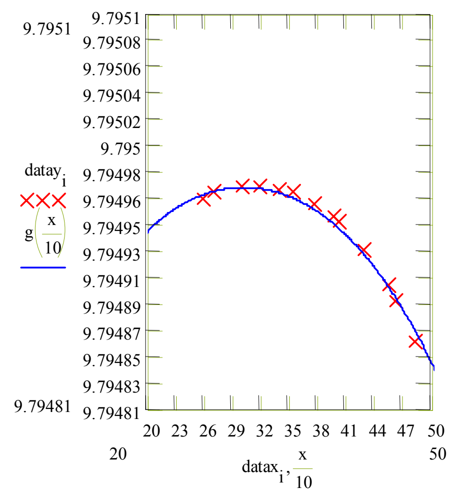
Compensated Sapphire Resonator Design

External Element Compensation (cont'd.)

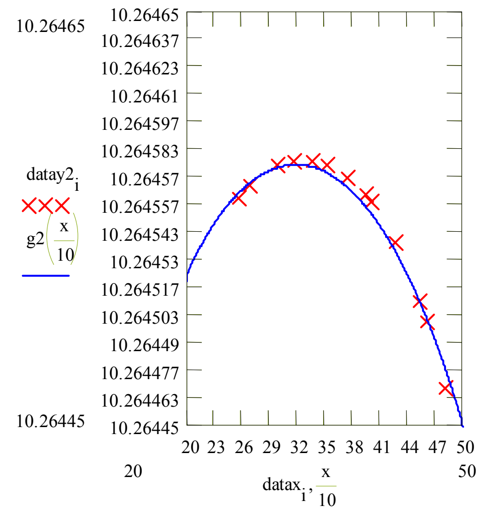
Paramagnetic Spins (Ruby)

0.03% Ruby Turnover Temperatures (TM Modes)

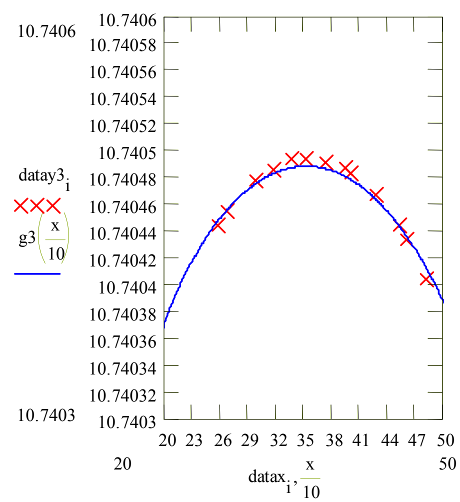
TM	Tt(K)	1/T	T^4(x10^-11)
11	30.10	-0.0031	-3.166
12	31.93	-0.0045	-3.354
13	35.07	-0.0076	-3.445
14	42.61	-0.0163	-2.901



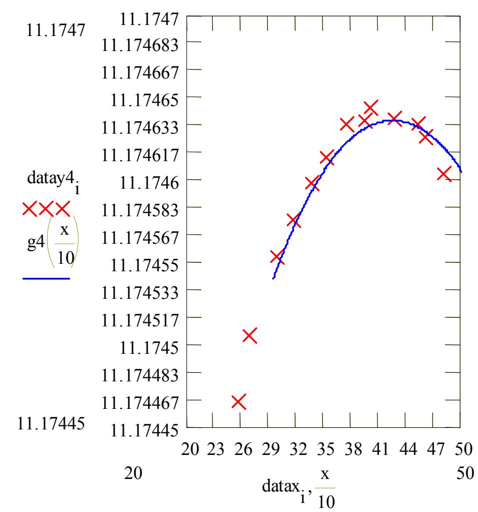
$WGH_{1,1,11}$



$WGH_{1,1,12}$



$WGH_{1,1,13}$



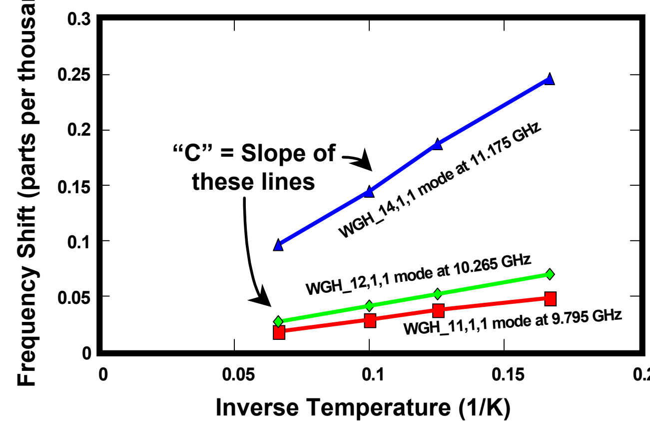
$WGH_{1,1,14}$

External Element Compensation (cont'd.)

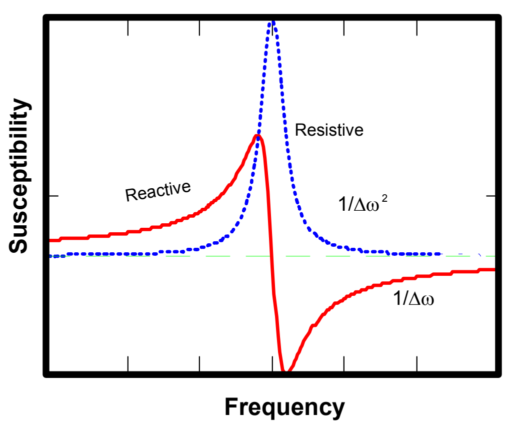
Paramagnetic Spins (Ruby)

- Verified low spin-dependent losses in ruby while allowing compensation of sapphire resonator at 10K
  - First simultaneous measurement of temperature tuning and Q in ruby
  - Confirm 1/T spin-dependent temperature tuning
  - Confirm Lorenzian frequency dependencies of both tuning and losses
  - Infer spin-dependent compensated resonator Q of 2 to $3x10^9$ with 0.03% Cr doping, RF design requires Q of only $2x10^8$
  - Confirm that only WGH modes couple to spins
  - Required ruby energy is < $1x10^{-3}$, allows substantially non-resonant coupling, insensitivity to configuration

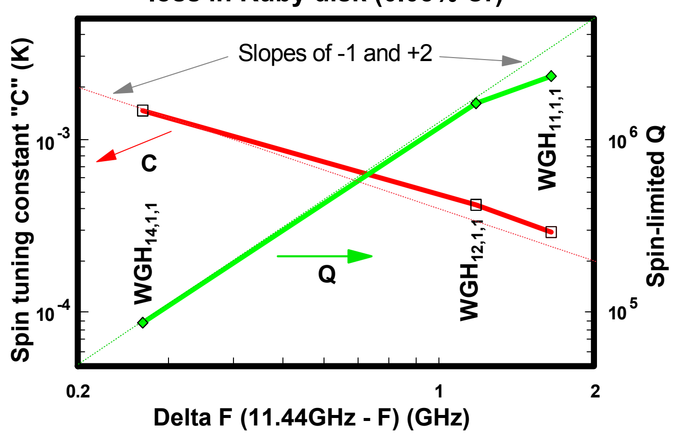
Spin-dependent frequency shift of 0.03% Ruby Resonator



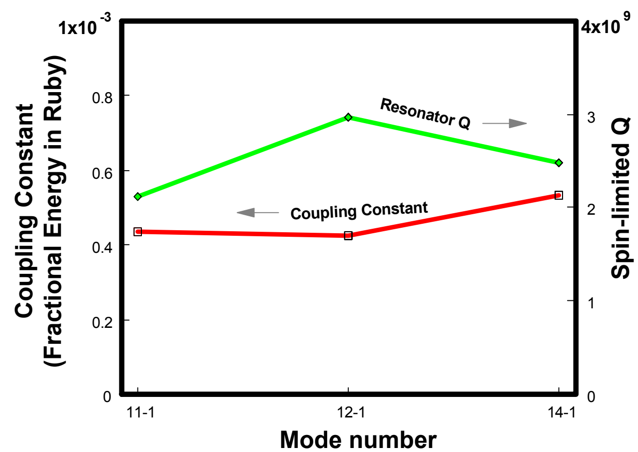
Expected Lorentzian line-shapes



Measured spin-dependent tuning and loss in Ruby disk (0.03% Cr)



Inferred coupling constant requirement and spin-limited resonator Q

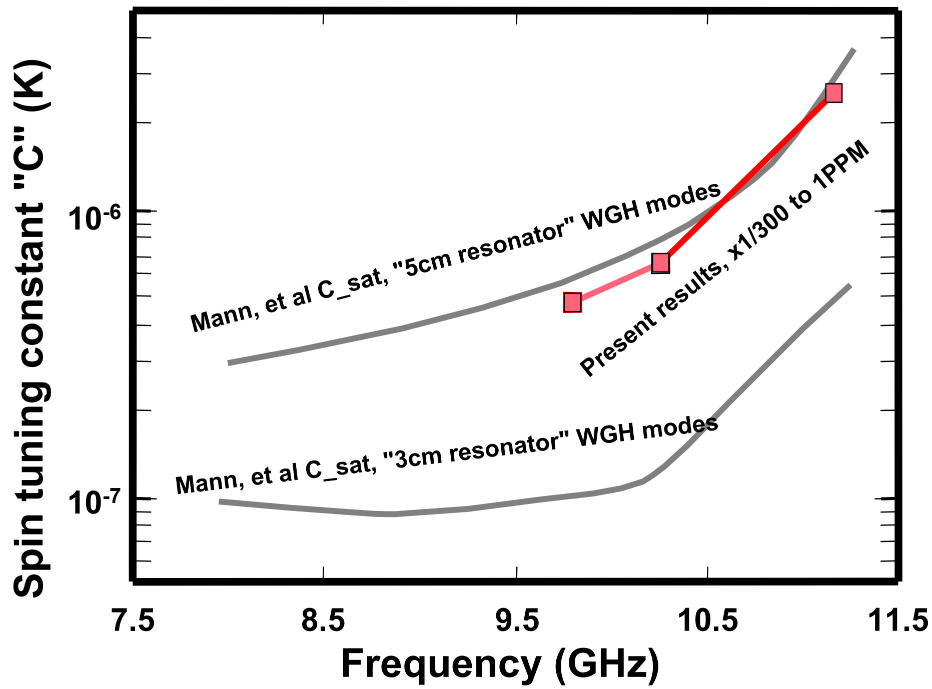


External Element Compensation (cont'd.)
Paramagnetic Spins (Ruby)

Comparison with other published results

- Our results show excellent agreement with those of Mann, et al, when scaled to the estimated 1PPM concentration of their "5cm" resonator

1/T frequency pulling scaled to 1PPM Cr compared with published data



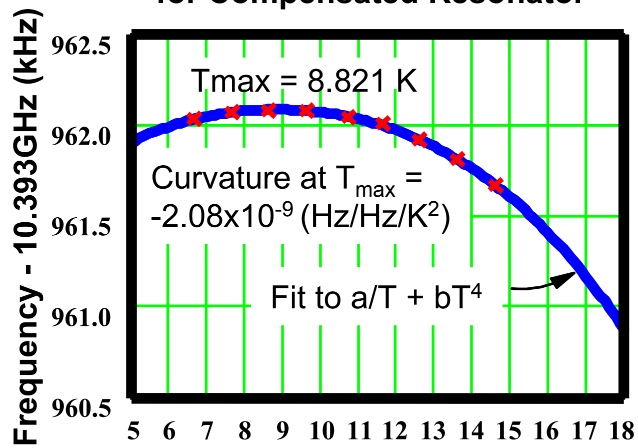
External Element Compensation (cont'd.)

Paramagnetic Spins (Ruby)

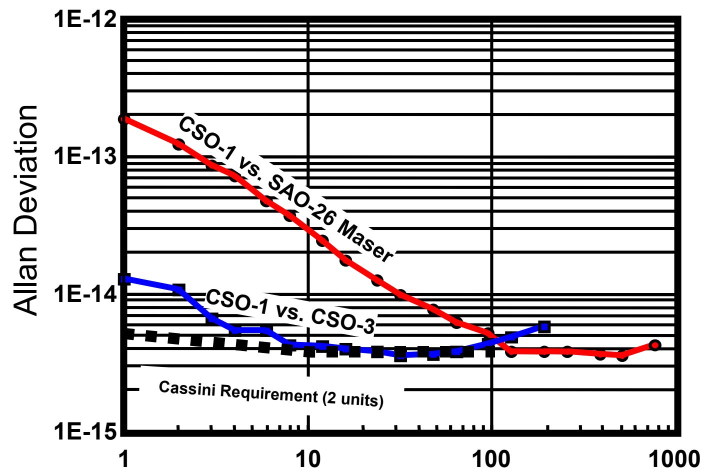
Compensated Operation

- **Measured turnover temperature in first ever compensated sapphire resonator with adjustable turnover below 40K**
  - Measured turnover at 8.821K compared to predicted 7.43K
  - Second assembly gave 8.54K
- **Compensation response time is ~ 0.75 sec**
  - Longer than 0.1 sec expected but allows reduction factor of 10 at 10 second measuring time and 100 at 100 seconds
  - Large thermal mass takes out short-term temperature fluctuations, can easily meet goals
- **Measured stability of $7x10^{-15}$**
  - First resonator assembly -- find afterwards sapphire has lowest Q of all of our samples
  - Short term stability follows H-maser ref
  - Floor not apparently due to thermal variation
    - 10^{-11}/dB rf power dependence is likely limit now, may need cryogenic rf level detection
    - Higher resonator Q may also improve floor

Temperature Dependence of Frequency for Compensated Resonator



Measured Frequency Stability

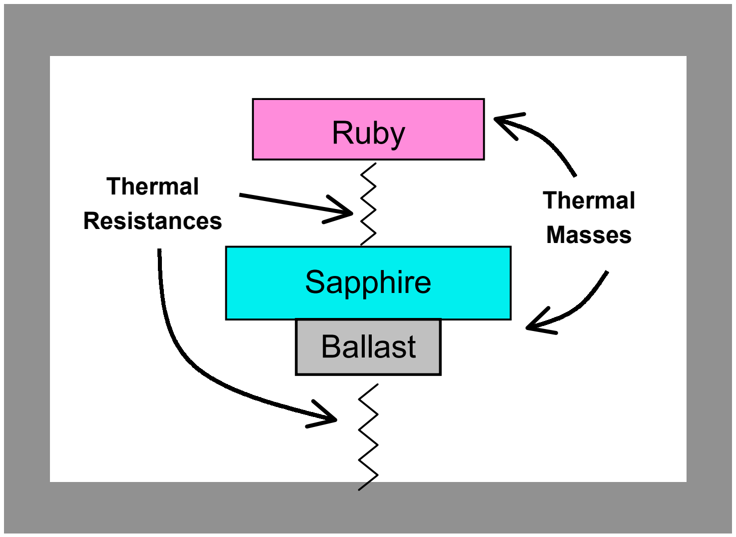


External Element Compensation (cont'd.)

Thermal Design

- Proper thermal design is crucial to the operation of a compensated resonator. Except for the case of resonator impurity compensation, the time constant of the compensation process can prevent the time-dependent variations in external temperature from actually being canceled. The problem is that the short-term temperature variations are then only imperfectly compensated.
  - Thus, if the time constant for the compensation process is 1 second and if temperature stability is 1 microkelvin at 1 second, this entire variation is essentially uncompensated. At 60K the sapphire's frequency variation with temperature is about 1 part in a million, and thus the stability of an oscillator based on this resonator would be $10^{-12} / \tau$ at best. This is a particular issue in cryocooled systems.
  - Temperature variations can be reduced by thermal ballast, adding a 1000 second thermal time constant between can and resonator can reduce this variation to 10^{-15} / tau, good enough for most applications.

External Element Compensation (cont'd.)
Thermal Design



Shielding Can

Typical time constant between Sapphire and Ruby is

$\tau_{SR} \sim 1.0$ seconds

And with thermal ballast between Sapphire and Can a time constant of

$\tau_{CS} \sim 3000$ seconds

can be achieved.

Even though the can is weakly attached to the sapphire, a fast change in the can's temperature gives rise to a fast (1 sec) change in the temperature difference between sapphire and ruby given by:

$$\Delta T_{SR} = (\tau_{SR} / \tau_{CS}) \times \Delta T_{CAN}$$

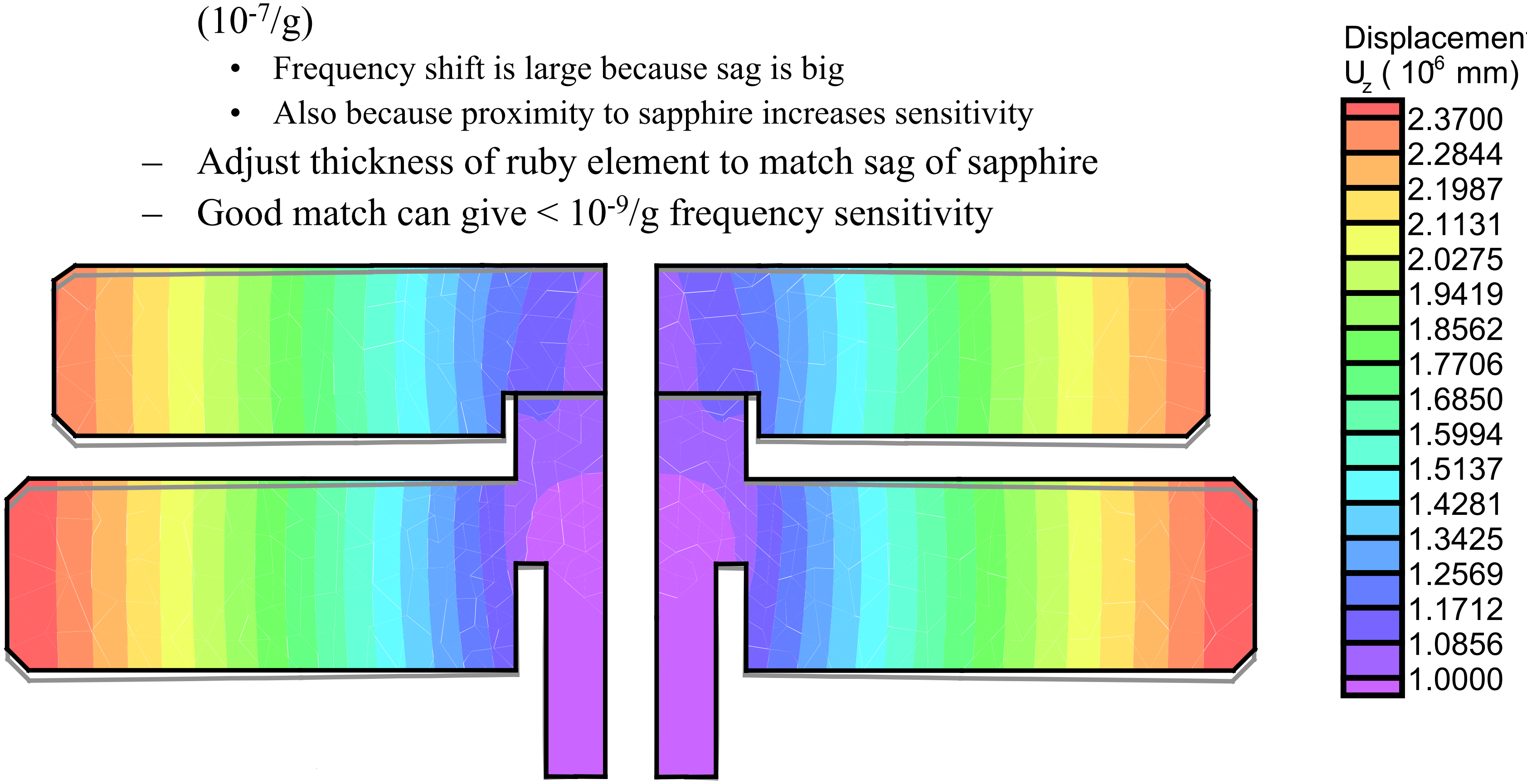
The sapphire resonator's temperature coefficient of frequency at 10K is approximately 2×10^{-9} and so a 1 mK change in can temperature changes the frequency by:

$$\Delta\omega/\omega = \Delta T_{SR} \times 2\times10^{-9} = (1.0/3000) \times .001 \times 2\times10^{-9}$$
$$= 7\times10^{-16}\ (/\text{mK})$$

External Element Compensation (cont'd.)

Acceleration Sensitivity

- Gravitational sag in sapphire and ruby elements matched by Mechanical F. E. calculation
- Plug in sag as displacement into Electromagnetic F.E. calculation to estimate gravitational sensitivity of composite structure
  - Thin (1mm) ruby disk would allow compensation but give large g sensitivity (10^{-7}/g)
    - Frequency shift is large because sag is big
    - Also because proximity to sapphire increases sensitivity
  - Adjust thickness of ruby element to match sag of sapphire
  - Good match can give < 10^{-9}/g frequency sensitivity

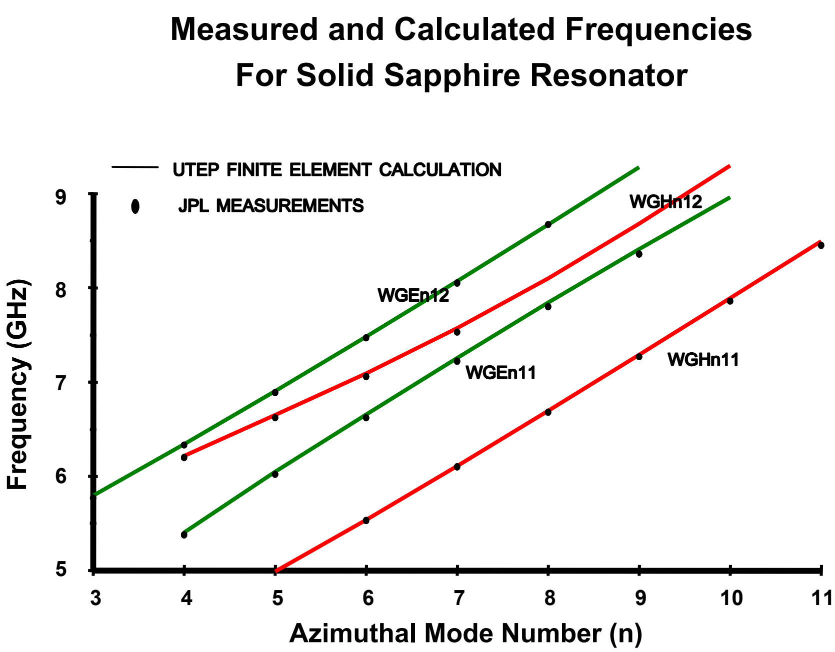


Calculation Methodologies - Boundary Value

- Access to accurate calculation of mode frequencies is crucial to analyzing and evaluating experimental results.
  - Resonators may show 50 or more modes in the frequency range of interest, and accurate calculation technique allows determination of modes without the difficulty of experimentally measuring mode field patterns, etc.
  - Accurate prediction of final frequency is a requirement for any end application.
  - Coupling strengths, frequency sensitivity to wall temperature depend in detail on the nature of the mode that is excited.
- Boundary value problem solution can be written in closed form for isotropic sphere--for other problems must expand solution as an infinite sum of basis functions which match Maxwell's equations in the various partial volumes, and also meet continuity conditions at the boundaries between regions.
- Several good approaches allow very accurate solutions for ring and disk geometries.
  - Disk or pill solutions give excellent agreement for lower order mode families (any n value), solving for most of the resonator space (Tobar and Mann).
- Ring geometry solutions that includes all resonator space show excellent agreement to 1 part in 1000 for the frequency of the WGH_{711} mode in a ring geometry, and good agreement with measured wall losses (Flory & Taber).

Calculation Methodologies – Finite Element

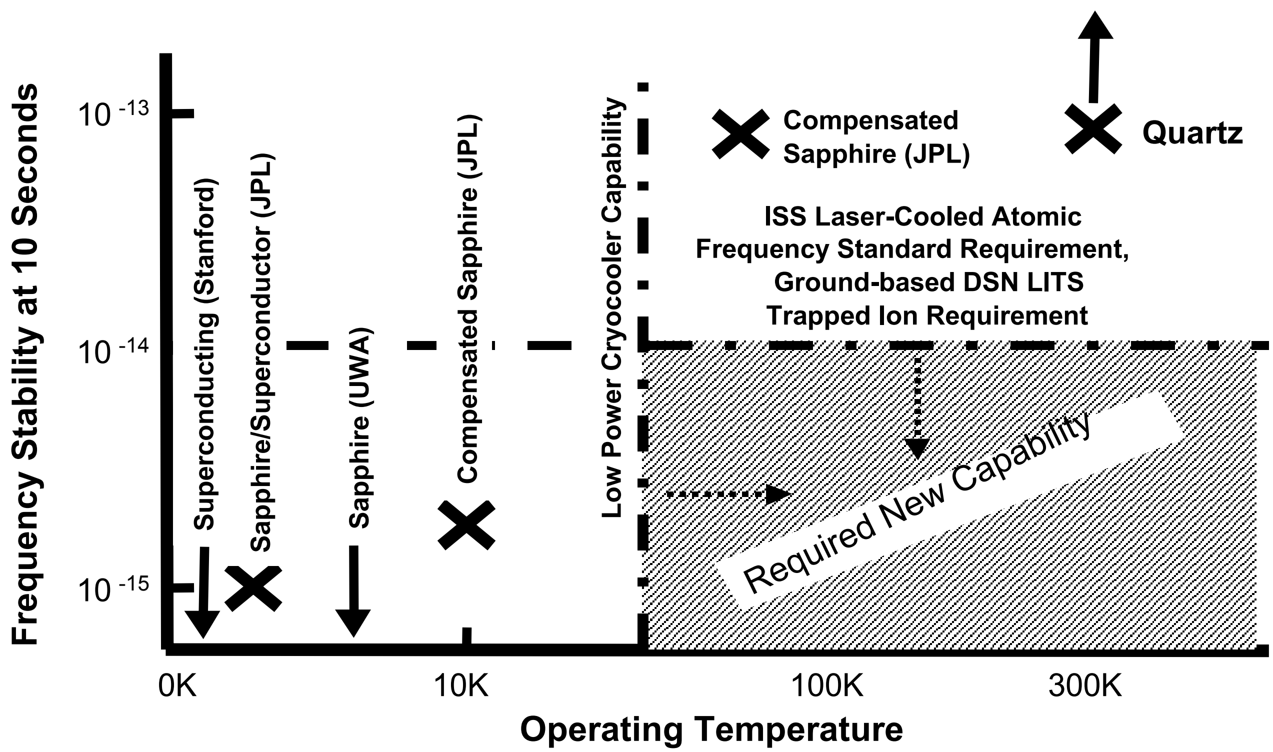
- Advantage of Finite Element approach is that it allows a uniform approach to new geometries--don't need to re-solve the boundary-value problem. Allows relatively complicated geometries.
- Whispering Gallery Modes pose difficult problem for Finite Element Methodologies.
  - Resonator has regions of widely disparate field magnitudes -- fields at the can wall are much smaller than at the sapphire, but both are important for successful design.



  - Number of nodes required for a 3-dimensional approach is prohibitive because of the large number of waves around the perimeter.
- Two-dimensional (cylindrically symmetric) approach is allowed by anisotropic sapphire if **c** axis is aligned with cylindrical axis. Software is now available.

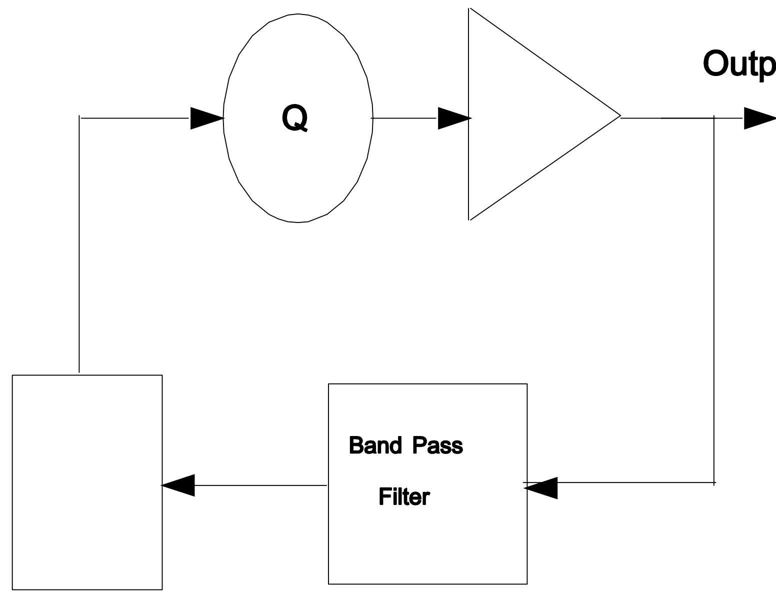
Future Prospects

- Present low-temperature sapphire oscillators show the highest available stability but are too expensive and/or inconvenient for use in many applications. If a capability could be developed that could reach stability of 10^{-14} or so at temperatures above 30K, the availability of small and inexpensive single stage cryocoolers would make this an attractive alternative in several applications. These include:
  - L.O. Applications for the new laser-cooled atomic frequency standards and for trapped ion frequency standards
  - Atomic standard cleanup oscillators for ultra-high short term stability--e.g. as for installation in DSN stations.
  - Wherever a quartz oscillator just isn't good enough



Active Frequency Sources

- Excellent phase noise performance for active sources to date a number of laboratories.
- Problem is high whispering-gallery mode density.
  - Approach is to use a band-pass filter to prevent oscillation in unwanted modes.
  - Also design details of resonator so that there are no nearby modes. Requires good analysis tools



- Somewhat more sophisticated designs have been applied at Liquid Helium temperatures in combination with a second high-Q tunable resonator

Frequency Sources with Passive Circuitry

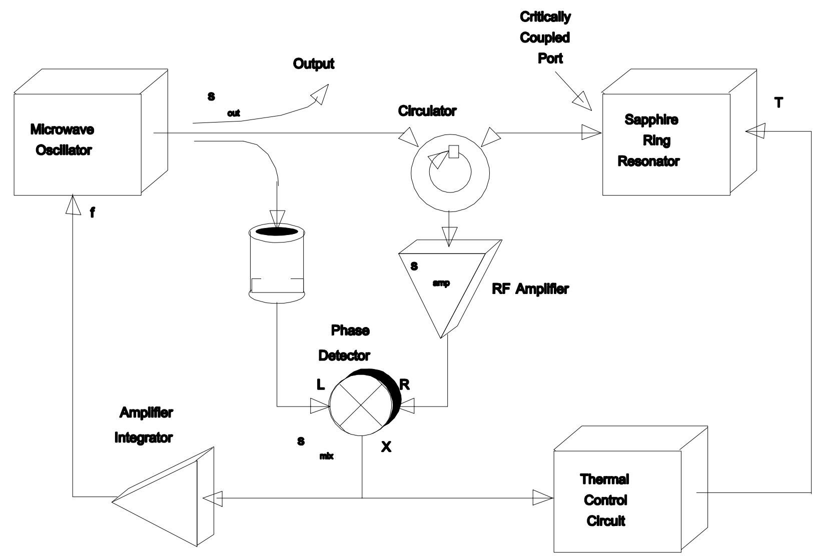
- Overmoded Whispering Gallery Resonator is good candidate for Passive Approach.
  - Large number of modes presents difficulty to Active approach -- spurious oscillations
  - Unwanted lower-Q modes also tend to be more strongly coupled-- this also exacerbates the spurious problem.

- AC Pound circuit is well suited for cryogenic applications
  - Lengths of microwave cables into the cryogenic environment vary as the temperature profile changes -- Pound circuit automatically compensates.
  - Long storage time of high-Q resonators allows relatively low frequency modulation -- Strongly satisfies condition $\delta\omega/\omega \ll 1$ required for successful Pound operation.

- Widely used for Helium-temperature operation.

Carrier Suppression Techniques

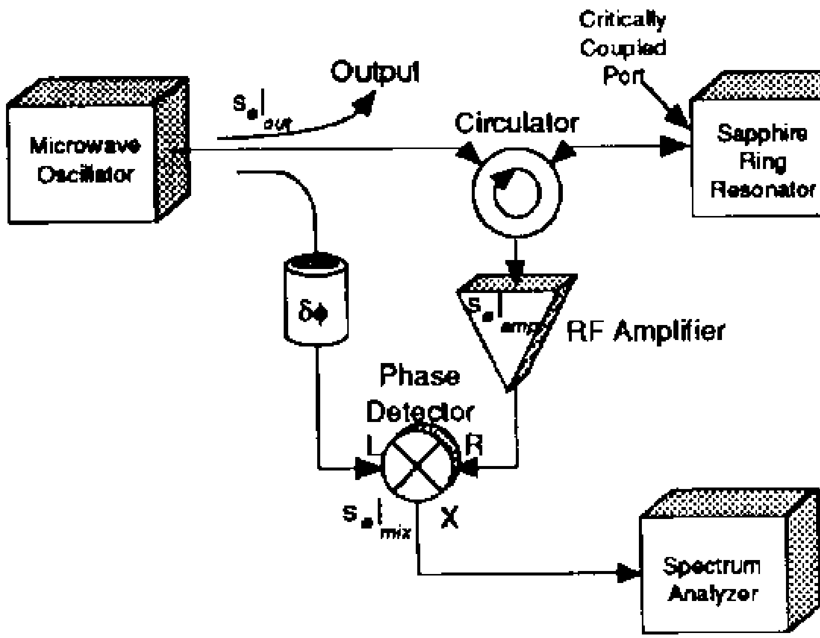
- Carrier suppression techniques now make possible a dramatic reduction in flicker noise. This has complemented the high Q of sapphire to allow unprecedented oscillator performance.
  - Passive sources already make possible very low phase noise.
  - Flicker noise in passive mixers is 20 to 30 dB lower than for microwave amplifiers required by an active oscillator.

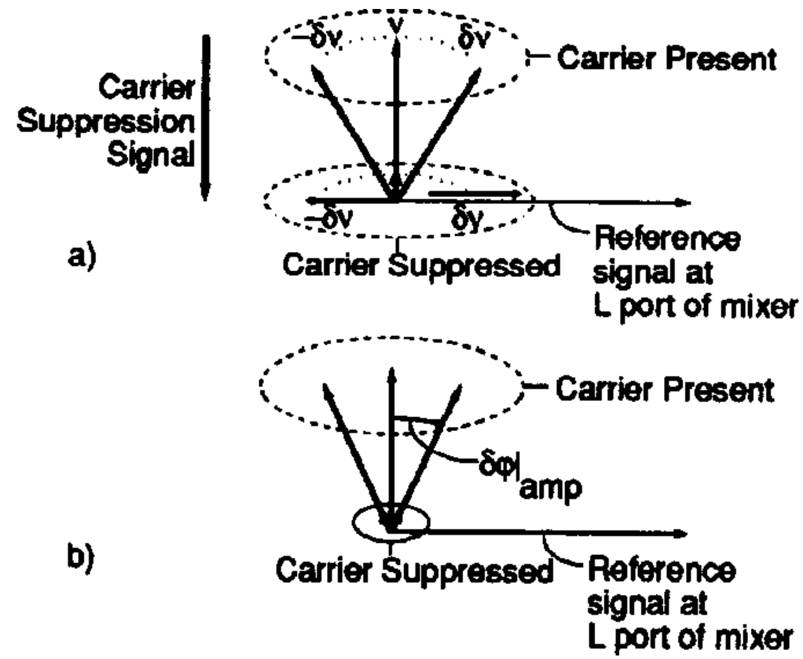


Block Diagram - Sapphire Phase Stabilizer (SPS)

Carrier Suppression Techniques (cont'd.)

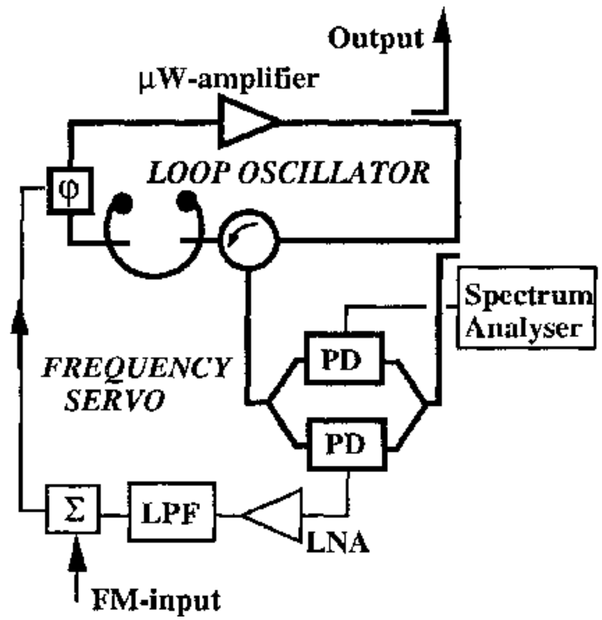
- Carrier suppression allows the use of an RF amplifier to further reduce mixer noise by suppressing the amplifier's own flicker noise which would otherwise ruin the performance.
  - Thus, 20 dB of amplifier gain allows a comparable reduction in mixer noise. However, since its own flicker noise is 20-30 dB higher than that of the mixer, 40-50 dB of carrier suppression is required to eliminate this amplifier noise .





Advanced Techniques

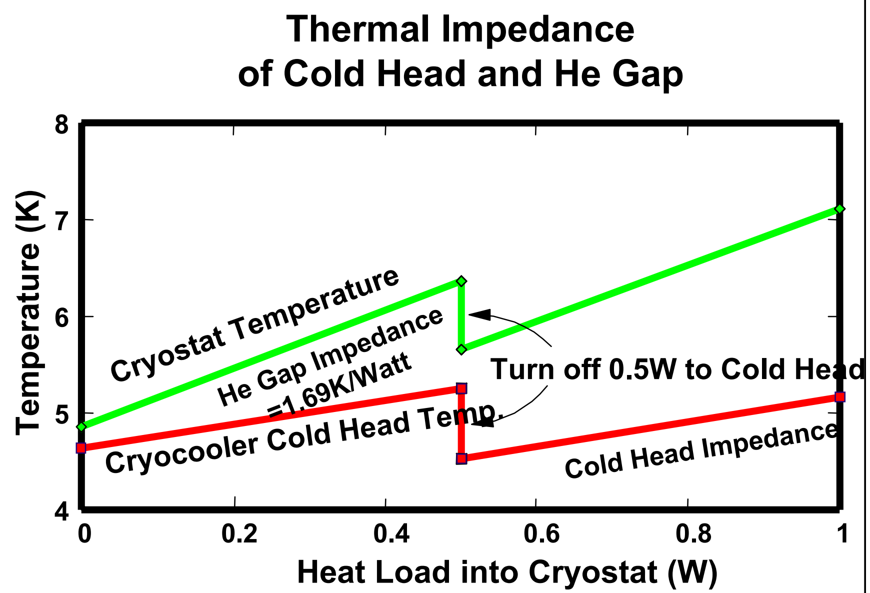
- A major development has been the recent application of carrier-suppression techniques to a variety of oscillator and test configurations. These often using path delay, or "interferometric" methodology to achieve the required cancellation.
  - Phase noise contributions of passive devices can now be verified for the first time to the very high accuracies required for ultra-low noise applications
  - Interferometric techniques have been applied to a variety of passive and active oscillator configurations. These now make possible ultra-low phase noise in room-temperature sapphire oscillators.
- Pound Circuits are required to achieve the highest possible stabilities
  - They complement phase reduction techniques by eliminating instabilities due to path lengths between electronics and the resonator.
  - Attention to and elimination of "false signals" is necessary to achieve the highest possible stability.
- Cross-correlation techniques can also be used to allow components for these oscillators to be tested to lowest-noise levels

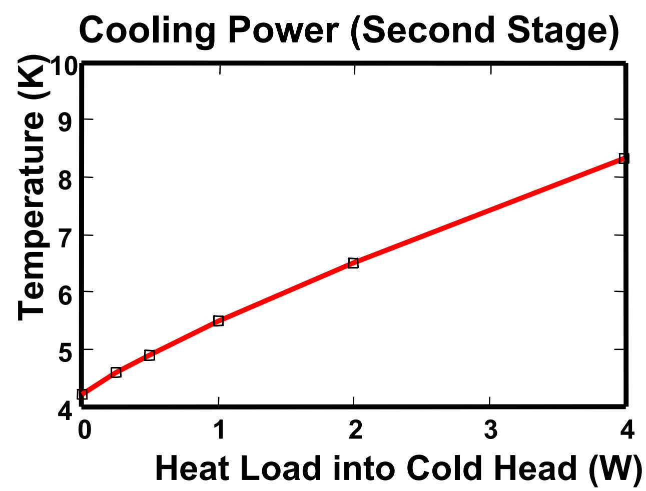


Loop Oscillator with Frequency Servo And Additional Microwave Readout Ivanov, Tobar, and Woode (see Ref.)

Cryogenics—10K CSO Cryocooler

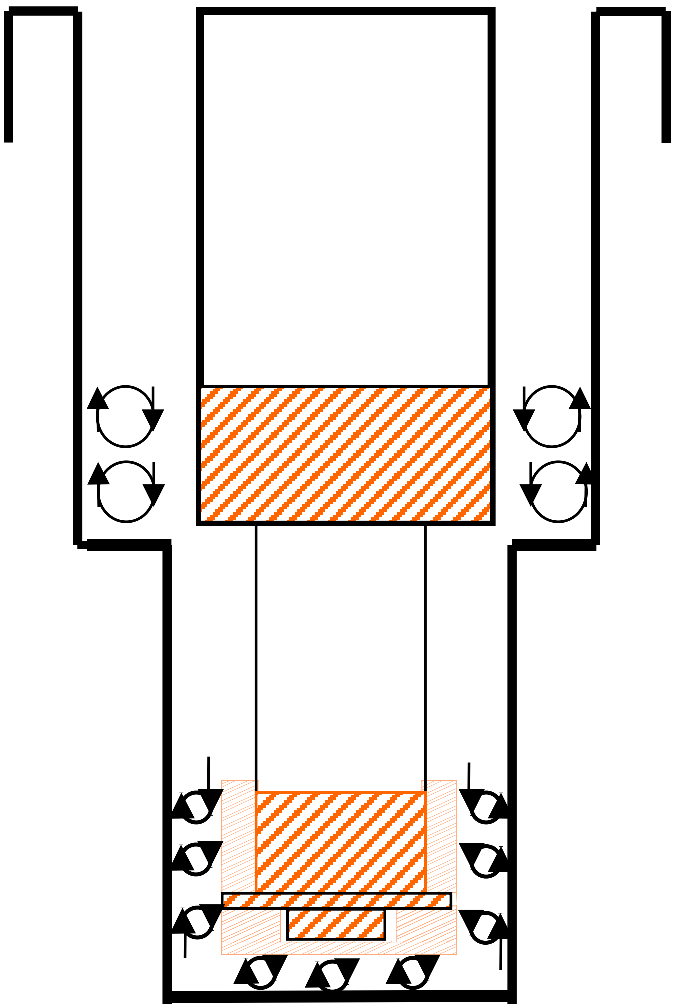
- Took delivery of cryogenic components
  - Closed-cycle refrigerator, compressor from Leybold/Balzers
  - Cryostat from Precision Cryogenics
- First test of cryogenic system
  - Three weeks continuous operation--no performance degradation with time
  - Verified temperatures and cooling power
    - Base temperature for 7K cryostat station is 5.45K with 250 mW added electric heat to simulate expected operational conditions
    - Measured temperature variation at 7K station is approximately 2mK p-p@2.5Hz cooler cycle frequency--much lower than expected 50mK p-p
    - Cooling power of cryocooler second stage verified--4.90K@0.5W heater input rising to 8.34K@4W input.
- Cool-down in 36 hours, warm up 48 hours





Cryogenics—10K CSO Cryocooler

CSO Helium Gas Heat Exchanger



Helium Heat Transfer

Vertical plate			one inch extension
gap(cm)	Ra	Nu	heat(watt)
0.2	1.87E+04	2.20	0.813
0.4	1.50E+05	3.88	0.620
0.6	5.05E+05	5.40	0.543
0.8	1.20E+06	6.84	0.498
1.1	3.02E+06	8.81	0.458
Horizontal plate			
gap(cm)	Ra	Nu	heat(watt)
0.2	1.87E+04	2.72	0.110
0.5	2.92E+05	5.09	0.082
1	2.34E+06	8.86	0.071
2.2	2.62E+07	18.97	0.068

Ra (Raleigh number) $Ra=\alpha g \Delta L^3/\kappa\nu$

Nu (Nusselt number) $Nu=H/(\kappa\Delta/L)$

α=isobaric thermal expansion coefficient

g=acceleration of gravity

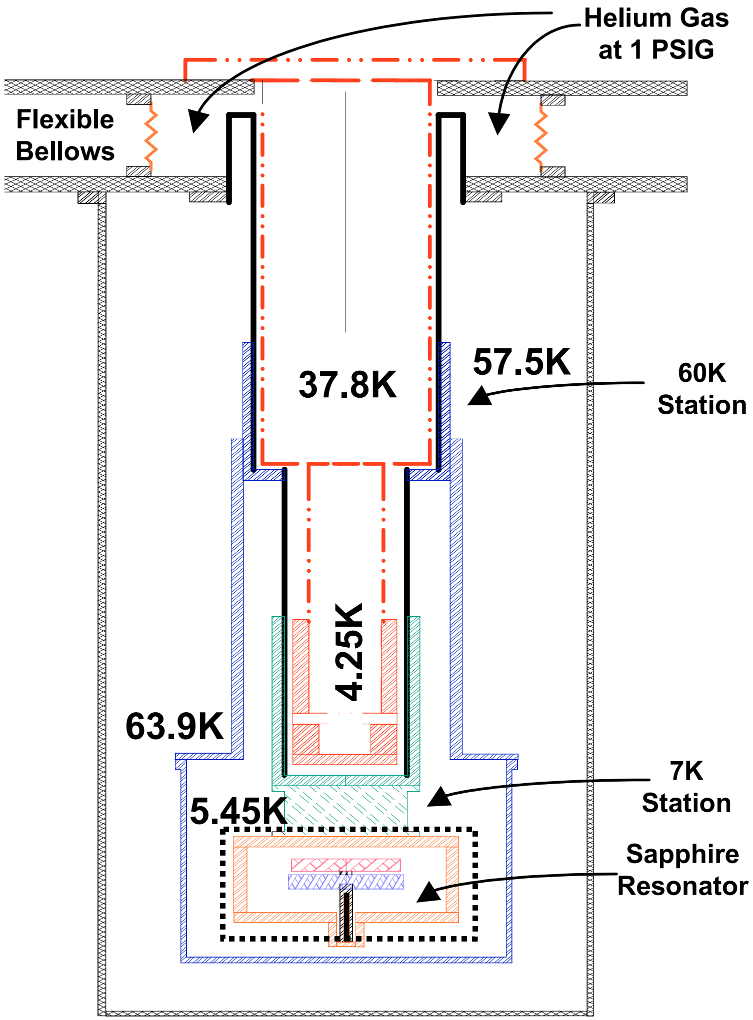
Δ=temperature drop between plates

L=cell height or gap space

κ=thermal diffusivity

ν=kinematic viscosity

Cryogenics—10K CSO Cryocooler



10K CSO Measured Temperatures
with 250mW into 7K Station

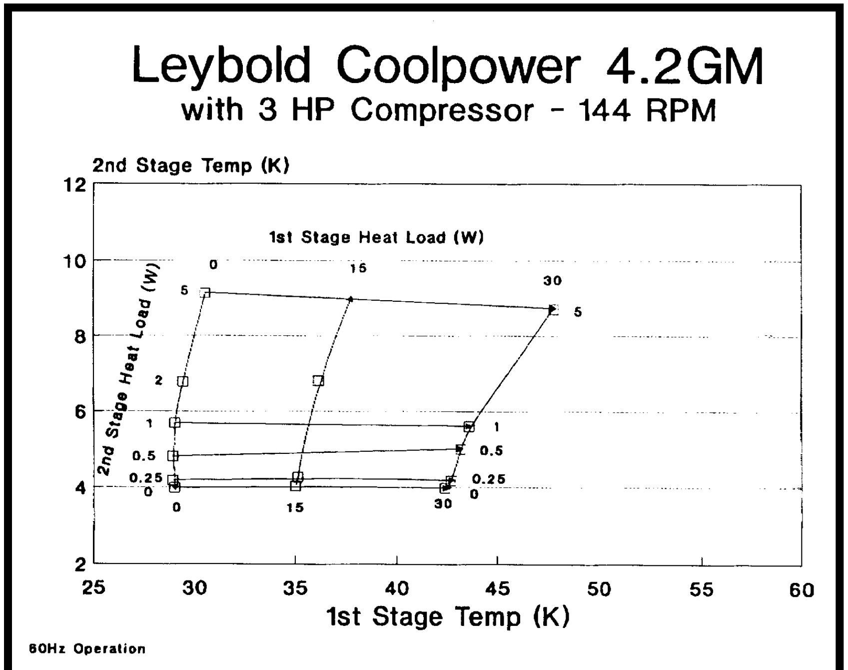
Thermal Design and Operation

Cooling Budget

Unit (Watts)

	Cold Head	Head Load	Designed Heat Flux
First stage @ 38 K	38	5.18	8.11
Second Stage @ 4 K	0.25	0.113	0.72

1/4 watt cooling at 4.2 Kelvin



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