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Optical gyroscope with whispering gallery mode optical cavities

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Abstract

We propose a miniature optical gyroscope based on a waveguide coupled sequence of whispering gallery mode microresonators. Sensitivity of this sensor is expected to be large due to large dispersion of the system.
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1. Introduction

Optical gyroscopes are widely used in spacecrafts and satellites, aircrafts and remote control devices, and in many other industrial and military sensors. Commercial applications of gyroscopes call for their minimum size, weight, low power consumption and integration to the optical fiber systems. The idea of usage of passive and active optical ring interferometers for detection of rotation was developed and implemented a couple of decades ago [1–3]. However, the further developments and improvements are still actual and important.

A miniature integrated optical sensors for gyroscope systems was recently proposed [4]. It was predicted that the sensor may possess high enough sensitivity even on a millimeter size scale. Another recent development was connected with highly dispersive media. It was noted that the phenomenon of "slow light" generated due to electromagnetically induced transparency and coherent population trapping may boost the sensitivity of optical gyros enormously [5] compared with the gyroscopes of the same size based on usual Sagnac effect [6] in dispersionless media.

Shortcomings in applications of atomic cells is connected with difficulty of their integration into fiber optics systems, specific operational frequencies determined by atomic transitions, and large residual absorption of light in atomic medium.

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There exists, however, an alternative to the "slow light" in atomic vapor that does not have those shortcomings. A sequence of coupled optical resonators can possess by large optical dispersion and slow group velocity of light [7]. Those systems were studied intensively with respect to optical soliton propagation, tunable optical delay lines, nonlinear optical switching and pulse compression, power limiting and others [8–10]. Recent successes with integration of whispering gallery mode resonators into photonic circuits prove practicality of those ideas [11–16].

We here propose to use high-Q microresonators coupled to an optical waveguide for the purpose of detection of a rotation. We discuss dispersion properties of the system and study the quantum restrictions of the sensitivity of an optical gyroscope based on "slow light" propagation in the system. We show that this device combines high sensitivity and small size, which is important for various practical applications.

2. An analysis of the response of a waveguide coupled to a resonator

We consider first a lossless ring resonator connected via a lossless coupler to an ideal transmission line. Energy transmission coefficient of the coupler T $(1 > T)$ determines the inverse value of the resonator finesse. The electromagnetic field $E_{\text{in}}(t)$ enters the resonator through the coupler, and $E_{\text{out}}(t)$ exits the resonator. We introduce the resonator field via two electromagnetic waves propagating inside the resonator and going out of the coupler $(E_1(t))$ and into the coupler $(E_2(t))$.

Assuming that the coupler has a zero response time, we can write the boundary conditions on the coupler surface as

$$E_1(t) = E_{\text{in}}(t)\sqrt{T} - E_2(t)\sqrt{1-T}, \tag{1}$$

$$E_{\text{out}}(t) = -E_{\text{in}}(t)\sqrt{1-T} - E_2(t)\sqrt{T}. \tag{2}$$

Fields E_1 and E_2 are connected by the condition

$$E_2(t) = -E_1(t-\tau), \tag{3}$$

where τ is the round-trip time for the resonator ($\tau = 2\pi a n_0/c$ for the case of a whispering gallery mode, where a is the mode radius, n_0 is the medium index of refraction, and c is the speed of light in the vacuum). Eq. (2) transforms to

$$E_1(t) - E_1(t-\tau)\sqrt{1-T} = E_{\text{in}}(t)\sqrt{T}. \tag{4}$$

Let us simplify the problem and consider the case when the field inside the resonator can be presented as a product of a fast oscillating part $\sim \exp(-i\omega_0 t)$ and a slow oscillating part $\tilde{E}(t)$, i.e. $E(t) = \tilde{E}(t)\exp(-i\omega_0 t)$. We assume that the slow field amplitude inside the resonator does not change significantly during the single round-trip time. Then the expression (3) can be expanded into Taylor series. Keeping linear term in τ only, we obtain

$$\tilde{E}_1(t-\tau) \simeq \tilde{E}_1(t) - \tau\dot{\tilde{E}}_1(t). \tag{5}$$

Substituting (5) into (1) we get an equation that allows us to calculate the field inside the resonator, if we know the pump field

$$\dot{\tilde{E}}_1(t) + \frac{1}{\tau}\left(1 - \sqrt{1-T}e^{i\omega_0\tau}\right)\tilde{E}_1(t) = \tilde{E}_{\text{in}}(t)\sqrt{\frac{T}{\tau^2}}. \tag{6}$$

Eq. (6) is quite general and it is valid for pulses longer than the resonator round-trip time, when we may consider a single resonator mode neglecting the others.

2.1. Resonant pulse

For a resonant pulse the carrier frequency ω_0 coincides with one of the resonant frequencies of the resonator, $\omega_0\tau = 2\pi N$ (N is an integer number). We consider the case of high finesse $1 \gg T$ resonator and long enough pulse so the pulse is not disturbed via the resonator dispersion. Than the solution of Eq. (6) is

$$\tilde{E}_1(t) = \frac{2}{\sqrt{T}}\tilde{E}_{\text{in}}\left(t - \frac{2\tau}{T}\right). \tag{7}$$

For the outgoing field, we find

$$\tilde{E}_{\text{out}}(t) = \tilde{E}_{\text{in}}\left(t - \frac{4\tau}{T}\right). \tag{8}$$

We see that the resonator results in the group delay of the pulse. The group delay is proportional to the light power build up in the resonator. Eqs. (7) and (8) are valid for pulses much longer than the group delay time $4\tau/T$, which also coincides with the width of the resonator resonance.

For the case of many resonators that interact with the waveguide, but do not interact with each other, the group delay increases additively [9]. The minimum group velocity in such a system may be estimated as

$$V_g|_{\min} \simeq \frac{aT}{2\tau} = c\frac{T}{4\pi n_0}. \quad (9)$$

The averaged group velocity does not depend on the resonator size.

Propagation of a light pulse through the region of the waveguide coupled to a resonator resembles usual "slow light" propagation: the front edge of the pulse is absorbed by the resonator and emitted in the propagation direction after ring-down time of the resonator. The back edge of the pulse is delayed in the same way. As the result the pulse shrinks in length during its interaction with the resonator. If initial pulse duration was t_0, the duration while pulse propagation through the resonator is $t_0 - 4\tau/T$. However, the pulse completely restores its shape after the interaction if the pulse is long enough, i.e. the frequency spectrum of the pulse is much narrower than the width of the resonator resonance.

2.2. Off-resonant pulse

For a pulse with carrier frequency obeying $\omega_0\tau = 2\pi N + \pi$ (N is an integer number) the energy build up in the resonator is small

$$\tilde{E}_1(t) = \frac{\sqrt{T}}{2}\tilde{E}_{\rm in}(t), \quad (10)$$

and the output field is not influenced by the resonator

$$\tilde{E}_{\rm out}(t) = \tilde{E}_{\rm in}(t). \quad (11)$$

The group velocity is equal to the phase velocity as one would expect.

Therefore, to achieve small group velocity on the microsphere-modified waveguide one should tune all resonators to a single frequency which is equal to the frequency of the laser used for the measurements.

In the following section, we show how high optical dispersion may result in increase of a response of an optical gyroscope that utilizes the dispersive medium.

3. Optical gyro in a dispersive medium

It is known that linear dispersion may significantly increase optical "dragging coefficient" and, therefore, media characterized by slow propagation of light pulses may be useful for detecting optical rotation [5]. In this section, we derive an expression for the relative phase shift for two light waves counter-propagating in a dispersive medium. We use simple classical arguments as of [2].

Let us consider two light waves propagating in a circle waveguide of radius R that rotates clockwise with small angular velocity Ω. The clockwise propagating light wave makes full circle in time t_+ and counter-clockwise in time t_-, where

$$t_\pm = \frac{n_\pm}{c}(2\pi R \pm \Omega R t_\pm), \quad (12)$$

where $n_\pm$ is the index of refraction of the medium that originates from the medium dispersion and Sagnac effect. We assume that the dispersion results from a narrow transparency resonance of arbitrary nature. The difference between travelling times t_+ and t_- is

$$\Delta t = t_+ - t_- \simeq 4\pi R\frac{n_0}{c}\left(\frac{\Delta n}{n_0} + \Omega R\frac{n_0}{c}\right), \quad (13)$$

where $n_\pm = n_0 \pm \Delta n_0$, $n_0 \gg |\Delta n|$, n_0 is the index of refraction of the motionless medium ($\Omega = 0$) at frequency ω_0.

To find Δn we note that due to the Sagnac effect the resonant frequency that characterizes the rotating medium changes with respect to the frequency of the light depending on the propagation direction of the light relative to the rotation direction

$$\omega_\pm = \omega_0\left(1 \pm \Omega R\frac{n_0}{c}\right). \quad (14)$$

We assume that the material possesses large positive dispersion $\partial n/\partial\omega \gg 1/\omega_0$; than

$$\frac{\Delta n}{n_0} \simeq \frac{\omega_+ - \omega_0}{n_0}\frac{\partial n}{\partial\omega} = \Omega R\frac{\omega_0}{c}\frac{\partial n}{\partial\omega}, \tag{15}$$

and

$$\Delta t = 4\pi R^2\Omega\frac{n_0}{c^2}\left(n_0 + \omega_0\frac{\partial n}{\partial\omega}\right). \tag{16}$$

The phase difference between the waves is (cf. Eq. (2.1) of [2])

$$\Delta\phi = \frac{2\pi c}{\lambda}\Delta t = \frac{8\pi^2R^2\Omega n_0}{\lambda V_g}, \tag{17}$$

where $V_g \simeq c/[\omega_0\partial n/\partial\omega]$ is the group velocity in the system, $\lambda = 2\pi c/\omega_0$ is the wavelength of the light in the vacuum.

It is easy to see from Eq. (17) that the signal phase shift of the gyro increases inversely proportional to the group velocity. Therefore, if the gyroscope is made of a waveguide coupled to many resonators, the sensitivity of such loop may be large.

4. A gyroscope based on a waveguide coupled to lossless optical microresonators

Shot noise limited sensitivity of a gyroscope may be found from expression $\Delta\phi > 2n_{out}^{-1/2}$, where n_{out} is the sum of photons exiting the waveguide. In accordance with Eqs. (9) and (17) the shot noise limited sensitivity of a gyroscope that uses a ring transmission line constructed of a waveguide coupled to whispering gallery mode resonators (see in Fig. 1) is

$$\Omega|_{min} > 2\left(\frac{c}{4\pi R n_0}\right)^2\frac{T}{\omega_0}\sqrt{\frac{\hbar\omega_0}{P_{out}t_M}}, \tag{18}$$

where P_{out} is the output light power (we assume equal output powers of the counterpropagating light waves $P_\pm = P_{out}/2$), t_M is the measurement time. This sensitivity is $4\pi/T$ times higher than the sensitivity of a conventional gyroscope that uses a usual waveguide sensor. This factor is characterized by the resonator finesse and it may be as large as 10^4 for highly oblate spheroids [18] and 10^6 for microspheres [17]. Hence, utilization of the resonator-modified waveguide may significantly increase sensitivity of the rotation measurements. On the other hand, similar enhancement of the measurement sensitivity was earlier found for a rotation sensor based on a passive ring resonator of radius R [1]. Therefore, usage of the resonator-modified waveguide is reasonable when it is impossible to obtain high-Q factors for such a large optical resonator. Moreover, we show in the following that the single resonator rotation sensor, not the sensor based on the resonator-modified waveguide, has the maximum sensitivity of the measurement, determined by the material absorption.

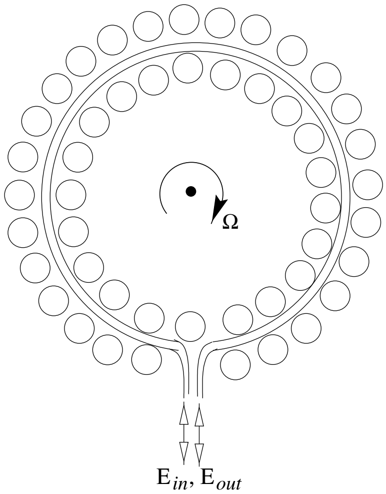

Fig. 1. A waveguide sensor for rotation measurements. High dispersion of the waveguide is achieved due to its coupling to high-Q WGM resonators placed along the waveguide rim.

Eq. (9) is valid for a resonator of any radius, but Eq. (18) was obtained under assumption that the radius of the resonator is much smaller than the radius of the waveguide. Direct substitution of this expression into Eq. (17) is not always possible. Deriving Eq. (17) we assumed that the rotation does not change properties of the ‘‘medium’’ the waveguide is fabricated from. This is true if the system includes point-like atoms. If the system includes resonators of a finite radius a, the rotation of the resonators influences the system response. The effect scales as a/R compared with the basic effect discussed above. If $R \gg a$, the influence may be neglected.

If ratio a/R is not small, the rotation of the resonators may act constructively as well as destructively on the measurement sensitivity. If the light propagates in the same direction in the loop and in the resonators, with respect to the loop rotation, the phase shifts due to resonator rotation and the waveguide loop rotation have the same sign. The resultant signal increases. It happens when the resonators are placed along the inner boundary of the waveguide loop. The phase shifts have the opposite signs when the resonators are placed outside of the circle.

Coupling of many resonators to the single waveguide and achieving the same resonant frequency for all of them is not an easy task. One of solutions here is producing resonators from germanate glass and subsequent permanent tuning of their spectra using ultraviolet radiation. It was shown that such a method allows for a mode frequency shift at the free spectral frequency range of a resonator while quality factor of the resonator stays intact [19,20].

5. A gyroscope based on a waveguide coupled to optical microresonators with losses

Presence of optical losses could significantly change infers made above, especially if we try to achieve maximum possible sensitivity of the gyro with real materials possessing finite absorption. Absorption is one of the main hindering mechanisms of the sensitivity increase of usual passive gyroscopes. The sensitivity increases with optical fiber length, however, it is impossible to use an optical fiber with infinite length, because the sensitivity depends also on the power of transmitted light. The maximum fiber length is determined by the absorption length of the material α^{-1}.

Absorption restricts the maximum sensitivity in our case as well. Because optical resonators could be made from the same material as the optical fiber connecting them, for example, fused silica, the average lifetime of a photon in such a resonator cannot exceed $\gamma^{-1} = n_0(\alpha c)^{-1}$. The maximum Q-factor of the resonator can be found from

$$Q = \frac{2\pi n_0}{\alpha\lambda}, \tag{19}$$

corresponding finesse $\mathscr{F}$ and factor T are

$$\mathscr{F} = (R\alpha)^{-1}, \quad T = 2\pi R\alpha. \tag{20}$$

Derived above equations for the gyro sensitivity should be modified to take the absorption into account. Transmission of a monochromatic electromagnetic wave of frequency ω through a resonator sidecoupled to a waveguide may be characterized by coefficient

$$T_{\mathrm{L}} = \frac{\gamma_c - \gamma - i(\omega - \omega_0)}{\gamma_c + \gamma + i(\omega - \omega_0)}, \tag{21}$$

where T_{L} is the transmission of light amplitude, γ, γ_c, and ω_0 are the linewidth originated from intrinsic resonator losses, linewidth due to coupling to a waveguide, and resonance frequency of a mode of the resonator (we assume that $|\omega - \omega_0|$ is much less than the resonator free spectral range). It is easy to see that the transmission is zero at the resonance and under condition of critical coupling ($\gamma_c = \gamma$) [21,22].

Eq. (8) should be modified accordingly

$$\tilde{E}_{\mathrm{out}}(t) \simeq \frac{\gamma_c - \gamma}{\gamma_c + \gamma}\tilde{E}_{\mathrm{in}}\left(t - \frac{4\tau}{T}\right), \tag{22}$$

where $T \simeq \tau(\gamma_c + \gamma)$. Transmission and dispersion for a long fiber (fiber length L is much bigger than R) coupled to the resonators could be estimated as

$$\tilde{E}_{\mathrm{out}}(t) \simeq \exp\left(-\frac{\gamma L}{\gamma_c R}\right)\tilde{E}_{\mathrm{in}}\left(t - \frac{2L}{\gamma_c R}\right), \tag{23}$$

which shows that for absorption $\exp(-1)$ maximum time delay is equal to $2\gamma^{-1} = 2n_0(\alpha c)^{-1}$ and, hence, is restricted by the properties of material only. Presence of a large number of resonators does not lead to the efficient increase of the delay time compared with a single resonator with maximum possible Q-factor. Fabrication of a large high-Q resonator can be a problem, and only then the resonator-modified waveguide is preferable.

Let us assume that we use a single whispering gallery mode resonator of radius R to detect the rotation. The sensitivity of the measurement based on phase change

$$\Omega > 2\left(\frac{c}{4\pi R n_0}\right)^2 \frac{2\pi R\alpha}{\omega_0} \frac{(\gamma_c + \gamma)^2}{\gamma|\gamma_c - \gamma|} \sqrt{\frac{\hbar\omega_0}{P_{in} t_M}} \qquad (24)$$

can be minimized at $\gamma_c = 3\gamma$.

6. Conclusion

We propose to use resonator-modified optical waveguides for fabrication of optical gyroscopes. We show that such a composite structure would allow for several orders of magnitude sensitivity enhancement that is particularly important for gyroscopes on a chip. The basic idea is based on the increase of the optical dragging and Sagnac effects in materials possessing large linear dispersion.

We also show that to achieve the maximum possible sensitivity of a passive rotation sensor it is better to use a large high-Q optical resonator of radius R instead of several resonators with the same Q and smaller size, coupled to a waveguide of the same radius R. This conclusion is a consequence of the presence of intrinsic absorption in realistic materials. If the absorption is neglected the sensors have nearly equivalent sensitivity.

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References

[1] G.A. Sanders, M.G. Prentiss, S. Ezekiel, Opt. Lett. 6 (1981) 569.
[2] W.W. Chow, J. Gea-Banacloche, L.M. Pedrotti, V.E. Sanders, W. Schleich, M.O. Scully, Rev. Mod. Phys. 57 (1985) 61.
[3] I.A. Andronova, G.B. Malykin, Phys. Usp. 45 (2002) 793.
[4] M.N. Armenise, V.M.N. Passaro, F. De Leonardis, M. Armenise, J. Lightwave Technol. 19 (2001) 1476.
[5] U. Leonhardt, P. Piwnitski, Phys. Rev. A 62 (2000) 055801.
[6] G. Sagnac, C.R. Acad. Sci. 157 (1913) 708.
[7] A.B. Matsko, O. Kocharovskaya, Y. Rostovtsev, G.R. Welch, A.S. Zibrov, M.O. Scully, in: B. Bederson, H. Walther (Eds.), Advances in Atomic, Molecular and Optical Physics, vol. 46, 2001, p. 191.
[8] J.E. Heebner, R.W. Boyd, Q. Park, Phys. Rev. E 65 (2002) 036619.
[9] J.E. Heebner, R.W. Boyd, Q. Park, J. Opt. Soc. Am. B 19 (2002) 722.
[10] J.E. Heebner, R.W. Boyd, Opt. Lett. 24 (1999) 847.
[11] D. Rafizadeh, J.P. Zhang, S.C. Hagness, A. Taflove, K.A. Stair, S.T. Ho, R.C. Tiberio, Opt. Lett. 22 (1997) 1244.
[12] B.E. Little, J.-P. Laine, D.R. Lim, L.C. Kimerling, H.A. Haus, Opt. Lett. 25 (2000) 73.
[13] J.-P. Laine, B.E. Little, D.R. Lim, H.C. Tapalian, L.C. Kimerling, H.A. Haus, Opt. Lett. 25 (2000) 1225.
[14] K. Djordjev, S.-J. Choi, S.-J. Choi, P.D. Dapkus, IEEE Photon. Technol. Lett. 14 (2002) 331.
[15] K. Djordjev, S.-J. Choi, S.-J. Choi, P.D. Dapkus, IEEE Photon. Technol. Lett. 14 (2002) 828.
[16] V.S. Ilchenko, A.A. Savchenkov, A.B. Matsko, L. Maleki, J. Opt. Soc. Am. B 20 (2003) 333.
[17] D.W. Vernooy, V.S. Ilchenko, H. Mabuchi, E.W. Streed, H.J. Kimble, Opt. Lett. 23 (1998) 247.
[18] V.S. Ilchenko, M.L. Gorodetsky, X.S. Yao, L. Maleki, Opt. Lett. 26 (2001) 256.
[19] A.A. Savchenkov, V.S. Ilchenko, T. Handley, L. Maleki, Opt. Lett. 28 (2003) 649.
[20] V.S. Ilchenko, A.A. Savchenkov, A.B. Matsko, L. Maleki, Proc. SPIE 4946 (2003) 195.
[21] Y. Xu, Y. Li, R.K. Lee, A. Yariv, Phys. Rev. E 62 (2000) 7389.
[22] A. Yariv, IEEE Photon. Technol. Lett. 14 (2002) 483.