Highly nondegenerate all-resonant optical parametric oscillator

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We show that a nondegenerate multifrequency parametric oscillator has different properties compared with the usual three-wave parametric oscillator. We consider, as an example, a scheme for a resonant cw monolithic microwave-optical parametric oscillator based on high-Q whispering gallery modes excited in a nonlinear dielectric cavity. Such an oscillator may have an extremely low threshold and stable operation, and may be used in spectroscopy and metrology. The oscillator mimics devices based on resonant $\chi^{(3)}$ nonlinearity and can be utilized for efficient four-wave mixing and optical comb generation. Moreover, the oscillator properties are important to better understand the stability conditions of long-baseline interferometers with movable mirrors that are currently used for gravity wave detection.

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I. INTRODUCTION

Optical parametric oscillators (OPOs) have been extensively studied since the discovery of lasers [1–3]. Properties of OPOs are well understood by now [4–6]. The cw-OPO is considered as an ideal device that can generate a broad range of wavelengths. Because of their reliability and excellent stability cw-OPOs are widely used, for example, in frequency chains [7,8], optical frequency comb generators [9–12], and for preparation of nonclassical states of light [13].

One of the most restrictive conditions for efficient parametric oscillations is the phase-matching conditions. Since the index of refraction in nonlinear materials strongly depends on the frequency, it results in the breakdown of the momentum conservation for pump and generated field photons propagating in a bulk material. Thus, periodically poled materials are often used to fulfill the phase-matching condition [4].

The parametric processes may be a useful approach in an important fundamental and practical problem, namely the coupling of fields with significantly different frequencies, such as a microwave field and light. However, the realization of phase matching for strongly nondegenerate parametric interactions is especially complicated. For example, the index of refraction of $LiNbO_3$ differs more than a factor of 2 for light and microwave fields. A particular solution of this problem was recently proposed and realized for tetrahertz optical frequency comb generators [10] and tetrahertz wave parametric sources [14,15], all for a planar geometry of the nonlinear crystal.

A possibility of phase matching for light confined in whispering gallery modes and microwaves was studied in Refs. [16–22]. Here, an efficient resonant interaction among optical whispering gallery modes and a microwave mode was achieved by engineering the geometry of a microwave resonator coupled to a dielectric optical cavity. To achieve the intended interaction, the optical cavity and the microwave resonator were pumped externally. The outgoing light was modulated as the result of the interaction.

In this work, we show that parametric interaction among waves with substantially different frequencies may significantly differ from the usual OPO behavior. We theoretically study two closely related examples: a nondegenerate OPO that converts light into light and microwaves, and an optical parametric process that converts light into light and mirror motion in long-baseline interferometers with moving mirrors [23,24].

Both examples involve an optical cavity that has a large number of nearly equidistant modes. Each pair of these modes may interact via the microwave field. As a result, the generation of two light fields with a frequency difference equal to twice the frequency of the microwave field is possible. This process resembles the four-wave mixing processes in $\chi^{(3)}$ media [6], where Stokes and anti-Stokes fields are generated from a single coherent pump field [25].

We also show that the system may be used for the generation of a comb of harmonics if a two-frequency optical pumping field is used. Unlike the usual comb generators based on the parametric interactions [9–12], our system does not need the application of a microwave field. The generation of harmonics occurs in a manner similar to harmonic generation in resonant $\chi^{(3)}$ media [26,27] and in stimulated Raman scattering in droplets with $\chi^{(3)}$ nonlinearity [28–31]. The two-frequency pumping field leads to the generation of the microwave field. The microwave field interacts with the pump and creates equally spaced harmonics. The process is most efficient when (i) the frequency difference for the two-frequency pump light corresponds to the resonant microwave frequency, and (ii) the spectrum of optical modes is equidistant. Both of these conditions may be satisfied in practice.

The optical properties of the strongly nondegenerate OPOs are similar to the properties of the usual atomic or molecular resonant structures. Indeed, our parametric system has narrow and stable resonances and can produce a four-wave mixing process, as well as generating a multiharmonic field. Hence, we might say that we have a model as an "artificial atomic structure." However, our system is not "nonlinear enough," it has no spontaneous emission, and, therefore, it has many differences compared to a "natural atom." Some of these similarities and differences are discussed in the paper.

Our theoretical analysis is rather general. As an example of its application to a realistic system we consider an OPO based on high-Q whispering gallery modes, and propose a

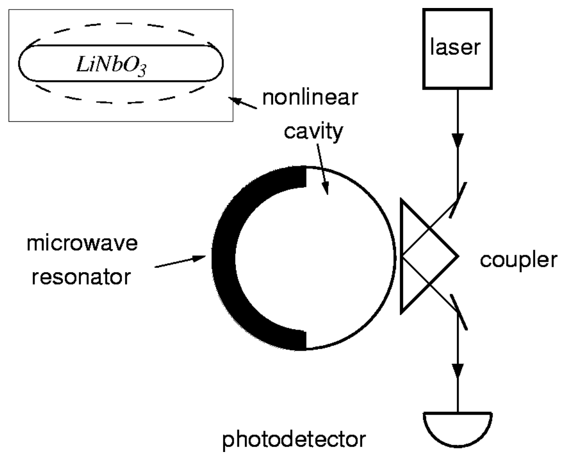

FIG. 1. Optical-microwave parametric oscillator. Inset: side view of the optical dielectric cavity. The boundaries of the cavity coincide with the boundaries of a spheroid, shown by dashed line.

configuration for a solid-state monolithic OPO that converts light into light and microwaves. We suggest to design the shape of the microwave resonator as in Ref. [18] and show that light modulation may appear without any microwave pumping. The microwave field is generated from vacuum as a result of the parametric interaction.

We assume that the pump laser radiation is introduced into a z-cut $LiNbO_3$ spheroid optical cavity via a coupling diamond prism (see in Fig. 1). The oblate spheroid cavity shape is essential to obtain a large free spectral range [17,32]. The optical cavity is placed between two plates of the microwave resonator. The resonant frequency of the microwave field can be adjusted to fit the frequency difference between the optical modes by changing the resonator shape. The spectrum of the dielectric cavity may be engineered by changing the profile of the index of refraction of the cavity material as well as the cavity shape [33].

Due to the $\chi^{(2)}$ nonlinearity of $LiNbO_3$, the modes of the microwave resonator and the optical cavity are effectively coupled. This coupling increases significantly for the resonant tuning of the fields due to the high quality factors of modes of optical cavity and microwave resonator, and the small mode volumes [34–37].

We show that with this OPO configuration the threshold of oscillation is as low as a few microwatts of light pump power for realistic parameters, and the stability of the signal may be better than that of the pump due to high quality factor of the whispering gallery modes. Therefore, this OPO holds both a promise for an efficient optical microwave modulator and may be used as a light source for optical frequency measurement and high precision spectroscopy, i.e., for atomic clocks [38,39], where small size of the device and low power consumption are important.

The problem of strongly nondegenerate parametric instability also arises in a long-baseline interferometer with suspended movable mirrors. The nonlinearity has a ponderomotive origin there. Such interferometers are currently used for the detection of gravity waves [23,24]. Here, in contrast to the case of an OPO mentioned above, a low threshold of the oscillation is a disadvantage because it substantially reduces the detection sensitivity. We show that for an equidistant spectrum of an interferometer, the threshold may be significantly increased. Ideally, if all parasitic modes are suppressed the interferometer might be stable even if the frequency of the oscillation of a mirror coincides with the free spectral range of the optical cavity.

The paper is organized as follows: In Sec. II, we review the main properties of the usual three-mode OPO. In Sec. III, we discuss the four-mode OPO. In Sec. IV, we study the conditions for the generation of a frequency comb in an all-resonant OPO with two-frequency optical pumping. In Sec. V, we discuss the stability of a cavity with moving mirrors.

II. DOWN-CONVERSION OF LIGHT INTO LIGHT AND MICROWAVES

Let us consider the nonlinear interaction of a coherent laser radiation, microwave field, and generated light radiation (pump, idler, and signal, respectively). The pump and the signal waves are nearly resonant with different modes of an optical nonlinear dielectric cavity, while the microwave field is nearly resonant with a mode of the microwave resonator coupled to the dielectric cavity. We assume that only two modes of light and a single mode of the microwave obey to the resonant condition.

The Hamiltonian describing this system is (see, for example, Ref. [40])

$$\hat{H}=\hbar\omega_a\hat{a}^\dagger\hat{a}+\hbar\omega_b\hat{b}^\dagger\hat{b}+\hbar\omega_c\hat{c}^\dagger\hat{c}+\hbar g(\hat{b}^\dagger\hat{c}^\dagger\hat{a}+\hat{a}^\dagger\hat{b}\hat{c}), \tag{1}$$

where ω_a and ω_b are the eigenfrequencies of the optical cavity modes, ω_c is the eigenfrequency of the microwave resonator mode, $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the annihilation operators for these modes respectively,

$$g=4\pi\omega_a\frac{\chi^{(2)}}{\epsilon_a}\sqrt{\frac{2\pi\hbar\omega_c}{\epsilon_c\mathcal{V}_c}}\left[\frac{1}{\mathcal{V}}\int_{\mathcal{V}}d\mathcal{V}\Psi_a\Psi_b\Psi_c\right] \tag{2}$$

is a coupling constant, $\chi^{(2)}$ is the electro-optic constant for the material of the dielectric cavity, $\mathcal{V}$ is the whispering gallery mode volume, $\mathcal{V}_c$ is the volume of the microwave field, Ψ_a, Ψ_b, and Ψ_c are the normalized dimensionless spatial distributions of the modes.

Using Hamiltonian (1), we derive equations of motion for the field operators:

$$\dot{\hat{a}}=-i\omega_a\hat{a}-ig\hat{b}\hat{c}, \tag{3}$$

$$\dot{\hat{b}}=-i\omega_b\hat{b}-ig\hat{c}^\dagger\hat{a}, \tag{4}$$

$$\dot{\hat{c}}=-i\omega_c\hat{c}-ig\hat{b}^\dagger\hat{a}. \tag{5}$$

We consider an open system. To describe such systems, appropriate decay and pumping terms should be introduced into Eqs. (3)–(5). The decay with necessity leads to leakage of quantum fluctuations into the system. To describe these fluctuations we use the Langevin approach [3].

Let us introduce slowly varying amplitudes for the mode operators as follows:

$$\hat{a}=Ae^{-i\omega_0 t},\quad \hat{b}=Be^{-i\omega_- t},\quad \hat{c}=Ce^{-i\omega_M t},\tag{6}$$

where ω_0 is the carrier frequency of the external pump of the mode $\hat{a}$, ω_-, and ω_M are the frequencies of generated light and microwaves, respectively. These frequencies are determined by the oscillation process and cannot be controlled from outside. However, there is a ratio between them,

$$\omega_0=\omega_-+\omega_M.\tag{7}$$

The equations for the slow amplitudes of the intracavity fields follow from Eqs. (3)–(5):

$$\dot{A}=-\Gamma_A A-igBC+F_A,\tag{8}$$

$$\dot{B}=-\Gamma_B B-igC^\dagger A+F_B,\tag{9}$$

$$\dot{C}=-\Gamma_C C-igB^\dagger A+F_M,\tag{10}$$

where

$$\Gamma_A=i(\omega_a-\omega_0)+\gamma,$$

$$\Gamma_B=i(\omega_b-\omega_-)+\gamma,$$

$$\Gamma_C=i(\omega_c-\omega_M)+\gamma_M.$$

F_A, F_B, and F_C are the Langevin forces, γ and γ_M are optical and microwave decay rates, respectively.

The Langevin forces are described by the following nonvanishing commutation relations:

$$[F_A(t)F_A^\dagger(t')]=[F_B(t)F_B^\dagger(t')]=2\gamma\delta(t-t'),\tag{11}$$

$$[F_C(t)F_C^\dagger(t')]=2\gamma_M\delta(t-t'),$$

and average values

$$\langle F_A\rangle=\sqrt{\frac{2\gamma W_A}{\hbar\omega_a}},\quad \langle F_B\rangle=\langle F_C\rangle=0,\tag{12}$$

where W_A is the pump power of the mode applied from outside. We assume that the fluctuations entering each mode from outside are in the coherent state and are uncorrelated with each other.

Let us solve the set (8)–(10) keeping expectation values only. Neglecting the optical saturation of the microwave oscillations, we obtain from Eqs. (8) and (10) in the steady state

$$\langle A\rangle=-i\frac{g}{\Gamma_A}\langle B\rangle\langle C\rangle+\frac{\langle F_A\rangle}{\Gamma_A},\tag{13}$$

$$\langle C\rangle=-i\frac{g}{\Gamma_C}\langle B^*\rangle\langle A\rangle.\tag{14}$$

Substituting Eqs. (13) and (14) into Eq. (9), we get

$$\langle\dot{B}\rangle+\langle B\rangle\left(\Gamma_B-\frac{g^2}{\Gamma_C^*}\frac{|\langle F_A\rangle|^2}{|\Gamma_A|^2}\left|1+\frac{g^2|\langle B\rangle|^2}{\Gamma_A\Gamma_C}\right|^{-2}\right)=0.\tag{15}$$

This equation has a nontrivial steady-state solution if the expression in parentheses is equal to zero. From the real and imaginary parts of this expression, we derive equations for the amplitude and the frequency of the generated field

$$\left|1+\frac{g^2|\langle B\rangle|^2}{\Gamma_A\Gamma_C}\right|^2=\frac{\gamma_M}{\gamma}\frac{g^2}{|\Gamma_C|^2}\frac{|\langle F_A\rangle|^2}{|\Gamma_A|^2},\tag{16}$$

$$\frac{\omega_b-\omega_-}{\gamma}=\frac{\omega_c-\omega_M}{\gamma_M}.\tag{17}$$

The expression for the oscillation threshold can be found using the assumption that the right-hand side term in Eq. (16) should exceed unity. Assuming the resonant tunings of all the fields $\Gamma_A=\Gamma_B=\gamma$, $\Gamma_C=\gamma_M$, introducing quality factors as $Q=\omega_0/(2\gamma)$ and $Q_M=\omega_M/(2\gamma_M)$, recalling $|F_A|^2/\gamma^2=4WQ/(\hbar\omega_0^2)$, and using Eq. (2); we derive an expression for the threshold value for the pump power

$$W_{th}=\left(\frac{1}{\chi^{(2)}}\right)^2\frac{\epsilon_a^2\epsilon_c}{128\pi^3}\frac{\mathcal{V}_c}{Q_M}\frac{\omega_0}{Q^2},\tag{18}$$

where we assumed that the normalized overlapping integral among the modes is equal to 1/2.

Let us estimate the threshold power. For realistic parameters of a dielectric whispering gallery mode cavity coupled to a microwave resonator [18] $Q=3\times10^7$, $Q_M=10^3$, and $\mathcal{V}_c=10^{-7}$ cm^3 we get $W_{th}\approx 1\,\mu$W.

Using Eq. (7) we rewrite Eq. (17) as

$$\omega_-=\frac{\omega_b+\dfrac{\gamma}{\gamma_M}(\omega_0-\omega_c)}{1+\dfrac{\gamma}{\gamma_M}}.\tag{19}$$

There is also a ratio between signal and idler amplitudes for the case when oscillations occur,

$$\frac{|\langle B\rangle|^2}{|\langle C\rangle|^2}=\frac{\gamma_M}{\gamma}.\tag{20}$$

Therefore, if $\gamma_M\gg\gamma$, the oscillation frequency is pulled to the center of the corresponding optical cavity resonance, and the photon number in the optical cavity exceeds the photon number in the microwave resonator. Otherwise the microwave frequency is pulled to the center of the microwave resonance.

Let us now calculate phase diffusion in the system. We represent the field operators in form

$$A=(|\langle A\rangle|+\delta A)e^{i\phi_A},\tag{21}$$

$$B=(|\langle B\rangle|+\delta B)e^{i\phi_B},\tag{22}$$

$$C=-i(|\langle C\rangle|+\delta C)e^{i\phi_C}, \tag{23}$$

where δA, δB, and δC describe the amplitude fluctuations, and ϕ_A, ϕ_B, and ϕ_C describe the phase fluctuations of the fields.

Keeping the linear fluctuation terms only, we derive an equation for $\phi_A-\phi_B-\phi_C$ from Eqs. (9) and (10). This equation shows that the evolution of the phase difference is stable, hence

$$\dot{\phi}_A-\dot{\phi}_B-\dot{\phi}_C=0. \tag{24}$$

On the other hand,

$$\frac{\dot{\phi}_B}{\gamma}-\frac{\dot{\phi}_C}{\gamma_M}=\frac{1}{\gamma}\left[\frac{\omega_- -\omega_b}{\gamma}\frac{F_B+F_B^\dagger}{2|\langle B\rangle|}+\frac{F_B-F_B^\dagger}{2i|\langle B\rangle|}\right] -\frac{1}{\gamma_M}\left[\frac{\omega_M-\omega_c}{\gamma_M}\frac{F_C-F_C^\dagger}{2i|\langle C\rangle|}+\frac{F_C+F_C^\dagger}{2|\langle C\rangle|}\right]. \tag{25}$$

Introducing phase diffusion coefficient as $\langle\phi^2\rangle-(\langle\phi\rangle)^2=2Dt$, and taking in mind that the output power of the signal and idler can be written as $W_{-out}=\hbar\omega_-^2|\langle B\rangle|^2/Q$ and $W_{M\,out}=\hbar\omega_M^2|\langle C\rangle|^2/Q_M$, we derive from Eqs. (24) and (25)

$$D_B=\frac{\gamma^2}{(\gamma+\gamma_M)^2}D_A+\frac{\gamma^2\gamma_M^2}{(\gamma+\gamma_M)^2}\frac{\hbar\omega_-}{W_{-\ out}}\left(1+\frac{(\omega_- -\omega_b)^2}{\gamma^2}\right), \tag{26}$$

$$D_C=\frac{\gamma_M^2}{(\gamma+\gamma_M)^2}D_A+\frac{\gamma^2\gamma_M^2}{(\gamma+\gamma_M)^2}\frac{\hbar\omega_M}{W_{M\ out}}\left(1+\frac{(\omega_M-\omega_c)^2}{\gamma_M^2}\right), \tag{27}$$

where D_A is the diffusion coefficient for the pump field. This coefficient is determined by the source of the pump. Because the quality factor of the whispering gallery modes may be very high (greater than 10^7), we are able to get a stable generation in our system.

III. UP- AND DOWN-CONVERSION OF LIGHT INTO LIGHT AND MICROWAVES

The above parametric interaction couples two light modes and a single microwave mode. The microwave field has a frequency nearly resonant with the frequency difference of the pump and the signal light. This picture is valid only if the optical modes are not equidistant, otherwise the pump light interacts with two optical modes having frequencies $\omega_{b\pm}\simeq\omega_a\pm\omega_c$. The condition for parametric oscillations drastically changes in this case.

The Hamiltonian describing this system is

$$\hat{H}=\hat{H}_0+\hat{V}. \tag{28}$$

$\hat{H}_0$ is the free part of the Hamiltonian

$$\hat{H}_0=\hbar\omega_a\hat{a}^\dagger\hat{a}+\hbar\omega_{b-}\hat{b}_-^\dagger\hat{b}_-+\hbar\omega_{b+}\hat{b}_+^\dagger\hat{b}_++\hbar\omega_c\hat{c}^\dagger\hat{c}, \tag{29}$$

where ω_a and $\omega_{b\pm}$ are the eigenfrequencies of the optical cavity modes, ω_c is the eigenfrequency of the microwave cavity mode, $\hat{a}$, $\hat{b}_\pm$, and $\hat{c}$ are the annihilation operators for these modes, respectively.

The interaction part of the Hamiltonian is

$$\hat{V}=\hbar g(\hat{b}_-^\dagger\hat{c}^\dagger\hat{a}+\hat{b}_+^\dagger\hat{c}\hat{a})+\text{adjoint}. \tag{30}$$

Instead of Eqs. (8)–(10), in this case we write

$$\dot{A}=-\Gamma_A A-ig(B_-C+C^\dagger B_+)+F_A, \tag{31}$$

$$\dot{B}_-=-\Gamma_{B-}B_--igC^\dagger A+F_{B-}, \tag{32}$$

$$\dot{B}_+=-\Gamma_{B+}B_+-igCA+F_{B+}, \tag{33}$$

$$\dot{C}=-\Gamma_C C-ig(B_-^\dagger A+A^\dagger B_+)+F_M, \tag{34}$$

where

$$\Gamma_A=i(\omega_a-\omega_0)+\gamma,$$

$$\Gamma_{B\mp}=i(\omega_{b\mp}-\omega_\mp)+\gamma,$$

$$\Gamma_C=i(\omega_c-\omega_M)+\gamma_M.$$

A, $B_\pm$, and C are the slowly varying amplitudes of the cavity mode operators; the optical (γ) and microwave (γ_M) decay rates as well as pumping forces F_A, $F_{B\pm}$, and F_M are introduced similarly to Eqs. (11) and (12).

Let us solve the set (31)–(34) in the steady state keeping expectation values only. From Eqs. (32) and (33), we get

$$\langle B_-\rangle=-i\frac{g\langle C^*\rangle\langle A\rangle}{\Gamma_{B-}},\quad \langle B_+\rangle=-i\frac{g\langle C\rangle\langle A\rangle}{\Gamma_{B+}}. \tag{35}$$

Substituting Eq. (35) into Eq. (34), we derive [cf. Eq. (15)]

$$\langle\dot{C}\rangle+\left(\Gamma_C-g^2|\langle A\rangle|^2\frac{\Gamma_{B+}-\Gamma_{B-}^*}{\Gamma_{B+}\Gamma_{B-}^*}\right)C=0. \tag{36}$$

This equation has a nontrivial steady-state solution if the expression in parentheses is equal to zero [cf. Eqs. (16) and (17)],

$$\gamma_M=\frac{g^2|\langle A\rangle|^2\gamma[(\omega_{b+}-\omega_+)^2-(\omega_{b-}-\omega_-)^2]}{[\gamma^2+(\omega_{b+}-\omega_+)^2][\gamma^2+(\omega_{b-}-\omega_-)^2]}, \tag{37}$$

$$\omega_c-\omega_M=\frac{\gamma_M}{\gamma}\frac{(\omega_{b+}-\omega_+)(\omega_{b-}-\omega_-)-\gamma^2}{(\omega_{b+}-\omega_+)+(\omega_{b-}-\omega_-)}. \tag{38}$$

Equation (37) determines the threshold of the parametric oscillations. It shows the power of the pump light that sustains the generation of a microwave field along with light sidebands. It is easy to see that no oscillations occur for any $\langle A\rangle$ if $(\omega_{b-}-\omega_-)^2\geqslant(\omega_{b+}-\omega_+)^2$. On the other hand, if $(\omega_{b+}-\omega_+)^2\gg(\omega_{b-}-\omega_-)^2$ we return to the case of usual parametric oscillator considered in the preceding section. Threshold pump power can be written as

$$\widetilde{W}_{th}=W_{th}\frac{[\gamma^2+(\omega_{b+}-\omega_+)^2][\gamma^2+(\omega_{b-}-\omega_-)^2]}{\gamma^2[(\omega_{b+}-\omega_+)^2-(\omega_{b-}-\omega_-)^2]}, \tag{39}$$

where we put $\Gamma_a=\gamma$, W_{th} is determined in Eq. (18).

The oscillation frequencies can be found from Eq. (38), if we take into attention that

$$\omega_{\pm}=\omega_0\pm\omega_M. \tag{40}$$

Analysis of Eq. (38) shows that the frequency of the microwaves is determined either by ω_c, if $\gamma\gg\gamma_M$, or by $\omega_{b+}-\omega_{b-}$, if $\gamma_M\gg\gamma$. There is also a ratio between signal and idler amplitudes similar to Eq. (20),

$$\frac{|\langle B_-\rangle|^2-|\langle B_+\rangle|^2}{|\langle C\rangle|^2}=\frac{\gamma_M}{\gamma}. \tag{41}$$

Let us consider, for instance, a nonlinear cavity with exactly equidistant spectrum

$$\omega_{b\pm}=\omega_a\pm\widetilde{\omega}_c, \tag{42}$$

where $\widetilde{\omega}_c$ is the frequency difference between the modes. The frequency of the microwave field ω_c is not necessarily equal to $\widetilde{\omega}_c$. Then $\widetilde{W}_{th}$ (18) is inversely proportional to product $(\omega_a-\omega_0)(\widetilde{\omega}_c-\omega_M)$. Therefore, there is no parametric process for any pump power if the pump is resonant with a cavity mode, $\widetilde{W}_{th}\rightarrow\infty$ if $|\omega_a-\omega_0|\rightarrow 0$. However, as is shown in the preceding section, the usual three-mode parametric process is the most efficient for the case of resonant pump tuning.

Let us consider now the case when $\gamma\gg\gamma_M$, $(\omega_{b+}-\omega_+)^2\gg(\omega_{b-}-\omega_-)^2$ $(|\langle B_-\rangle|^2\gg|\langle B_+\rangle|^2)$ and find the phase diffusion coefficient for beat note for modes B_+ and B_-. To do this, we introduce amplitude and phase fluctuations similar to Eqs. (21)–(23):

$$A=(|\langle A\rangle|+\delta A)e^{i(\phi_A+\varphi_A)}, \tag{43}$$

$$B_{\pm}=(|\langle B_{\pm}\rangle|+\delta B_{\pm})e^{i(\phi_{B\pm}+\varphi_{B\pm})}, \tag{44}$$

$$C=(|\langle C\rangle|+\delta C)e^{i(\phi_C+\varphi_C)}, \tag{45}$$

where $\varphi_\xi=\langle\xi\rangle/|\langle\xi\rangle|$ is the expectation value of the field phase.

Using this approximation, we derive the following expressions that connect phases of the pump and the generated fields (cf. Ref. [3]):

$$\dot{\phi}_A=\dot{\phi}_{B-}+\dot{\phi}_C=\dot{\phi}_{B+}-\dot{\phi}_C, \tag{46}$$

$$\dot{\phi}_C\simeq\frac{\omega_M-\omega_c}{\gamma_M}\frac{F_C-F_C^\dagger}{2i|\langle C\rangle|}+\frac{F_C+F_C^\dagger}{2|\langle C\rangle|}.$$

The phase diffusion of the microwave field $\phi_{B+}-\phi_{B-}=2\phi_C$ has a diffusion coefficient

$$D\simeq 4\gamma_M^2\frac{\hbar\omega_M}{W_{M\,out}}\left(1+\frac{(\omega_M-\omega_c)^2}{\gamma_M^2}\right). \tag{47}$$

Such parametric oscillations show some similarity with the standard four-wave interaction in media with the third-order nonlinearity. For example, they have much in common with near resonant four-wave mixing produced in atomic vapors [25]. (i) Namely, in those experiments Stokes and anti-Stokes optical fields are generated spontaneously from vacuum, the same is expected in our case; (ii) the frequency difference between anti-Stokes and pump fields and pump and Stokes fields is equal to the hyperfine splitting of the ground state of rubidium atoms, the frequency difference is determined by the frequency of the microwave field in our case; (iii) the threshold of the oscillations in atomic medium is a few microwatts for the pump power, the same level of the threshold pump power is expected in our case; (iv) in atomic experiments, oscillation becomes possible due to long-lived atomic coherence, in our case the role of atomic coherence is played by the microwave mode; (v) phase diffusion of the beat note of the Stokes and anti-Stokes fields generated in the atomic system is determined by the atomic coherence lifetime [41], in our case it is determined by the quality factor of the microwave mode (47).

However, there are differences between resonant four-wave mixing in atomic media and in our system: (a) the atomic medium has essentially a nonlinear response that leads, in particular, to the creation of a "dark state" that does not interact with the multifrequency light [42], there is no such state in our nonlinear system; (b) the Stokes and anti-Stokes fields have nearly the same amplitudes in atomic medium. In our case the fields have different amplitudes.

IV. OPTICAL COMB GENERATION

Optical comb generation can be achieved using an electro-optic modulator with external microwave pumping [9,10]. The resonant atomic and molecular systems may lead to efficient generation of a comb of optical frequencies without such pumping [26,27]. It is also known that whispering gallery modes result in the enhancement of Raman scattering [28–31]. We now study the possibility of comb generation in our resonant parametric system.

Let us assume that our system consisting of a nonlinear oblate spheroid microcavity and a microwave resonator is pumped by a two-frequency light. Each mode of the pump is resonant with a mode of the cavity, and the frequency difference for the pump is equal to the resonant frequency of the microwave resonator. We assume also that the cavity modes are equidistant and the frequency difference between them is equal to the microwave frequency. This is true in the first approximation for the spheroid [32]. However, even the residual dispersion can be compensated [33].

Under such conditions the two optical fields generate a microwave field in our system. The microwave field interacts with the light and the interaction generates an equidistant frequency spectrum. This process is closer in similarity to the comb generation in an atomic medium [26,27] than to the usual comb generation technique, where a microwave field applied to a nonlinear crystal modulates light [9,10].

To describe the comb generation, we write the interaction Hamiltonian as

$$\hat{V}=\hbar g\sum_{n=-\infty}^{\infty}(\hat{a}_{n-1}^{\dagger}\hat{c}^{\dagger}\hat{a}_n+\hat{a}_{n+1}^{\dagger}\hat{c}\hat{a}_n)+\text{adjoint}, \quad (48)$$

where $\hat{a}_n$ is annihilation operator for nth cavity mode. We assume that modes are completely identical with respect to their quality factor and coupling to the microwave field.

Using Eq. (48), we derive equations of motion for the modes. For the sake of simplicity we consider the case of exact resonance for all the modes. In slowly varying amplitude and phase approximation, equations for the expectation values of the field amplitudes are

$$\dot{A}_n=-\gamma A_n-ig(A_{n-1}C+C^*A_{n+1})+F_n(\delta_{n,0}+\delta_{n,-1}), \quad (49)$$

$$\dot{C}=-\gamma_M C-ig\sum_{n=-\infty}^{\infty}(A_{n-1}^*A_n+A_n^*A_{n+1}), \quad (50)$$

where F_n stands for the pumping of the modes, $\delta_{i,j}=1$ if $i=j$ and $\delta_{i,j}=0$ if $i\neq j$. In other words, we assume that only modes with $n=0$ and $n=-1$ are pumped.

We introduce the function

$$A(\theta)=\sum_{n=-\infty}^{\infty}A_n e^{i\theta n}, \quad (51)$$

and present the amplitude of the microwave field as $C=|C|\exp(i\phi_C)$. Then rewriting Eq. (49) in steady state as

$$A_n=-i\frac{g}{\gamma}(A_{n-1}C+C^*A_{n+1})+\frac{F_n}{\gamma}(\delta_{n,0}+\delta_{n,-1}), \quad (52)$$

multiplying each of them with $\exp(i\theta n)$ (n corresponds to the index of term γA_n), and summarizing them over all n, we derive

$$A(\theta)=\frac{F_0+F_{-1}e^{-i\theta}}{\gamma+2ig|C|\cos(\theta+\phi_C)}. \quad (53)$$

The solution for each mode A_n can be written as

$$A_n=\frac{1}{2\pi}\int_0^{2\pi}A(\theta)e^{-i\theta n}d\theta. \quad (54)$$

Equation (53) contains unknown constants $|C|$ and ϕ_C. To find them we write Eq. (50) in the steady state

$$C=-i\frac{g}{\gamma_M}\sum_{n=-\infty}^{\infty}(A_{n-1}^*A_n+A_n^*A_{n+1}) \quad (55)$$

and substitute there A_n (52). This gives us $\phi_C=-\pi/2+\arg(F_{-1}^*F_0)$ and equation for $|C|$,

$$\begin{aligned}|C|&=\frac{g}{\gamma\gamma_M}\sum_{n=-\infty}^{\infty}|A_{n-1}^*F_n+F_n^*A_{n+1}|(\delta_{n,0}+\delta_{n,-1})\\&=\frac{2g}{\pi\gamma_M}|F_{-1}^*||F_0|\int_0^{2\pi}\frac{\sin^2(\theta+\phi_C)d\theta}{\gamma^2+4g^2|C|^2\cos^2(\theta+\phi_C)}\\&=\frac{4g}{\gamma_M}\frac{|F_{-1}^*||F_0|}{4g^2|C|^2}\left(\sqrt{1+\frac{4g^2|C|^2}{\gamma^2}}-1\right).\end{aligned} \quad (56)$$

The solution of this equation yields the amplitude of the microwave field.

Finally, we note that the actual time-dependent amplitude of the light can be written as

$$A(t)=e^{-i\omega_0 t}\sum_{n=-\infty}^{\infty}A_n e^{-i\omega_M nt}, \quad (57)$$

where ω_0 is the carrier frequency of the mode with $n=0$. Exchanging θ with $-\omega_M t$ in Eqs. (53) and (54), we derive

$$A(t)=\frac{F_0e^{-i\omega_0 t}+F_{-1}e^{-i(\omega_0-\omega_M)t}}{\gamma+2ig|C|\cos(\omega_M t-\phi_C)}. \quad (58)$$

Let us note here that the signal generated in our system is different from the usual phase modulated signal. This occurs because of the saturation of the oscillation.

The width of the frequency comb is determined by value $g|C|/\gamma$. To have a wide spectrum this value should be comparable with unity. Assuming that both pump harmonics have the same power W_A and, therefore, $|F_{-1}|\approx|F_0|$ we get

$$\frac{g^2}{\gamma\gamma_M}\frac{|F_{-1}||F_0|}{\gamma^2}=\frac{W_A}{W_{th}}\geqslant 1,$$

where W_{th} is the threshold power for the parametric oscillation (18). As we discussed above, this power can be as low as a few microwatts for realistic conditions. Therefore it is possible to generate a broad frequency comb in our system using a small pump power.

V. MULTIMODE REGIME OF PONDEROMOTIVE PARAMETRIC INSTABILITY

It was shown recently that long-baseline gravitational wave detectors may suffer from parametric instability [23]. This instability arises from ponderomotively mediated coupling between the mechanical oscillations of suspended cavity mirrors and the probe light that is used for detection of the mirrors' signal shift. The effect is undesirable because it might create a specific upper limit for the energy stored in the cavity. The sensitivity of the detection increases with the light power and, hence, such a process might pose an upper limit on the measurement sensitivity.

Since the planned circulating power in the interferometer is about 1 MW, it was shown that the power exceeds the threshold for parametric oscillations by almost a factor of 300 for realistic conditions, if the optical mode spectrum is

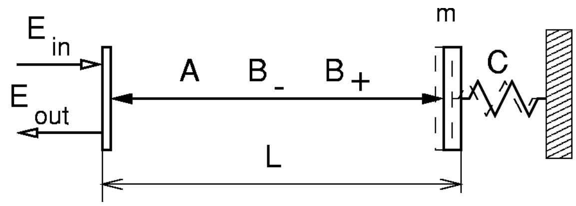

FIG. 2. Fabry-Perot cavity with movable mirror. Parametric instability is possible due to ponderomotive nonlinearity.

not equidistant (three-wave description of the parametric process is valid) [23].

We can easily describe the ponderomotive parametric instability using the technique presented above. In fact the interaction between the mirrors' mechanical oscillations and optical modes can be characterized by either Eqs. (8) and (9) or Eqs. (31)–(34), where C describes mechanical oscillations and the coupling factor g is appropriately chosen.

Let us consider a Fabry-Perot interferometer with one movable mirror that has mass m and mechanical resonance frequency ω_c (Fig. 2). The distance between the mirrors of the resonator is equal to L. Then the coupling constant between the mechanical degrees of freedom of the mirror and optical modes is

$$g=\frac{\omega_a}{L}\sqrt{\frac{\hbar}{2m\omega_c}}. \tag{59}$$

In the case of a nonequidistant mode spectrum, we may consider only two optical modes and a mechanical mode and use Eqs. (8) and (9) that gives us the result of Ref. [23]. The threshold power for the oscillation for resonant tuning of the pump laser follows from

$$\frac{2W_{th}}{\gamma}\frac{2QQ_M}{m\omega_M^2L^2}=1. \tag{60}$$

In the case of nearly equidistant modes we should use Eqs. (31)–(34). Then the threshold power increases according to Eq. (39). Since for long-base interferometers the main longitudinal mode spectrum is almost equidistant, we might expect that the threshold of the parametric oscillation will increase significantly. A problem may arise, however, due to transverse modes of the system. Therefore, to understand if the system is stable one needs to consider exact mode structure of a particular system.

VI. CONCLUSION

We have shown that a strongly nondegenerate multifrequency parametric oscillator possesses different properties as compared with the usual three-wave OPO. As an example of such an oscillator, we have studied a scheme for an all-resonant optical-microwave parametric oscillator based on whispering gallery modes excited in a nonlinear dielectric optical cavity and in a long-base cavity with moving walls.

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