Search for New Physics with Atomic Clocks

Lute Maleki and John Prestage

Quantum Sciences and Technology Group, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 USA

Abstract. We will discuss the physical ramifications, and describe an experiment with three high-precision clocks flying to within six solar radii of the sun for a test of a possible variation of the fine structure constant α. Measurement of the drift in ratios between the frequencies generated by each clock will probe for the variation of α. Since the response of each element to a change in α has a specific signature, this measurement will provide specific and unambiguous results. The sensitivity of this experiment to a changing α exceeds the sensitivity of recent tests based on observational astronomy, as well the geophysical bounds on α variations. Thus, the experiment will provide a compelling test of the standard model and the alternative theories.

1 Introduction

Recent developments in both theoretical and observational fronts have fueled a great deal of interest in a search for a variation of the fine structure constant. On the observational side, Webb et al. [1] have found evidence for a cosmological variation of the fine structure constant through an analysis of the absorption lines in galactic halos from quasar-emitted light. Their results indicate that the fractional change in α, averaged over redshift in the range of $0.2 \leq z \leq 3.7$ is $(-0.57 \pm 0.10) \times 10^{-5}$. On the theoretical side, many of the outstanding issues confronting fundamental physics, such as the failure to include gravity in the standard model, and puzzles of cosmology, such as inflation and the apparent accelerated rate of the expansion of the universe, appear to imply the existence of massless, or nearly massless scalar fields. These fields appear as dilaton or moduli in the M-theory, supporting the unification of gravity with other forces, as well as suggesting a possible breakdown of the Equivalence Principle. They also appear as quintessence in models of cosmology aimed at resolving fine tuning and other outstanding problems, such as a nonzero cosmological constant [2]. The scalar fields in these models imply a spatio-temporal variation of constants of nature, such as fine structure and other field coupling constants.

Despite these important developments, at this writing there is no clear consensus amongst researchers regarding the validity of the theoretical predictions, and the observational conclusions are not regarded as inconvertible. The question of if, how, and why the fine structure constant varies remains an open one.

It is clear then that a controlled experiment with sufficient measurement sensitivity beyond the current capabilities will be enormously important in clarifying some of the questions associated with α variations. SpaceTime is a space mission study aimed at providing such an experiment. It is based on flying an

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instrument based on three clocks that run on ground state hyperfine transitions of three different singly ionized atoms to within six solar radii of the Sun. The "tri-clock" instrument of SpaceTime is capable of testing a variation of α with four orders of magnitude more sensitivity, as compared with the results of quasar observations. As discussed below, the choice of the atomic clocks as the instrument was made to ensure that the results would be conclusive and free of many questions that have confronted previous investigations searching for a varying α.

At this point it is worthwhile to consider some of the consequences of a varying fine structure constant. The fine structure constant has been a point of fascination with physicists since it was introduced, and named, by Sommerfeld in 1916 as a useful constant in spectroscopy; it is a measure of the doublet structure of hydrogen and other atoms with a single valence electron. Sommerfeld also considered α as an indication of an intimate relation between charge and quantum. In the years following Sommerfeld's introduction of α, various physicists, starting with Eddington, have considered the relation between α and other constants of nature. This interest was also fueled by suggestive numerology that relates specific functions of π to the value of α.

The conjecture of varying fundamental constants has also a relatively long history and dates back to Dirac's "Large Number Hypothesis", which was based on the notion that there exists an underlying relationship between constants of nature, as manifested by large numbers, on the order of 10^{39}, that could be obtained by arranging them in various combinations [3]. Other *ad hoc* conjectures similarly have pointed to possible variation of constants, especially the gravitational constant G, through which a variation of α may also arise. These models, nevertheless, were all generally qualitative, and more importantly, lacked any observational support. The picture has changed in the last few years. Since a change in α implies a changing e, the charge of the electron, or c, the speed of light, or Planck's constant, h, through $\alpha = e^2/c\hbar$, several models based on variations of any of these dimensional constants have been devised [4–8]. There is, however, a good bit of controversy regarding the validity of these models, and if their predictions do or do not support [9,10] a violation of the Equivalence Principle, as well.

Atomic clocks have traditionally been used to test the prediction of general relativity. The first such test performed in 1976 by NASA's Gravity Probe A, where the rate of a hydrogen maser clock on a rocket in a sub-orbital trajectory was compared to that of a similar clock on the Earth's surface [11]. This measurement verified the exact prediction of a clock shift by general relativity to a part in 10^4, a precision that still stands unchallenged today. In a recent investigation it was shown that it is also possible to search for a variation in α by comparing the rate of drift of two clocks based on hydrogen and mercury ion [12]. This is because the energy of the hyperfine transition in atoms, which forms the basis for microwave clocks, have an αZ dependence, where Z is the atomic number. This first laboratory attempt to search for a varying α set a limit of $\sim 4 \times 10^{-14}$ per year for its temporal variation. This approach has recently been extended to the comparison of a rubidium and a cesium fountain clock, both based on microwave transitions [13], as well as the comparison of a cesium fountain with an optical mercury ion clock, where an optical transition in the ion was used [14]. These more recent experiments set the limit for a varying α a to be less than about 10^{-15}/yr. This is a less stringent limit than that obtained with an analysis of neutron capture rate applied to a natural thermonuclear reaction that occurred 1.5 billion years ago in Oklo mine, Africa [15], which places the limit on α variation to be less than 5×10^{-17}/yr. SpaceTime's instrument is designed to provide sensitivity to a variation in α at the level of 10^{-20}/yr by searching for any spatial dependence of α.

For alkali atoms, an expression for the hyperfine interval may be obtained, as follows:

$$A_s = \frac{8}{3}\alpha^2 g_I Z \frac{z^2}{n^{*3}} \left(1 - \frac{d\Delta_n}{dn}\right) F(\alpha Z)(1-\delta)(1-\epsilon)\frac{m_e}{m_p} R_\infty c \,. \qquad (1)$$

Here, z is the net charge of the ion without the valence electron, and n^* is the effective quantum number with $\Delta n = n - n^*$, δ and ϵ are related to the corrections for finite size of the nucleus. Thus the sensitivity of different clocks, based on atoms of different Z, to a change in the fine structure constant display specific signatures. In particular, the Casimir correction factor, $F(\alpha Z)$, (for the relativistic wave equation of the electron) leads to the differential sensitivity in the alkali microwave hyperfine clock transition frequencies f,

$$f = \alpha^4 \frac{m_e}{m_p} \frac{m_e c^2}{h} F(\alpha Z) \,. \qquad (2)$$

It is clear from the above equation that different atomic systems with different Z display different frequency dependencies on a variation of α through the αZ dependent terms. A direct test for a time variation of α can then be devised through a comparison of two clocks, based on two atomic species with different atomic number, Z.

This is a key feature of the SpaceTime instrument that in conjunction with the individual sensitivity of each atomic species to an α variation, can produce clear and unambiguous results.

Since the changing α in all model predictions is mediated by coupling of a scalar field to matter, the fall in the $1/R$ potential near the Sun will allow a direct test of the general relativity, where only the tensor field is allowed, and where the constants are not allowed any variation. This is an important point to consider in clock tests, and other tests searching for an α variation based on a signature of the failure of the equivalence principle (EP). Since EP is currently tested at about the 10^{-12} level [16] with no violations found, any test searching for α variations must have a sensitivity higher than 10^{-12} to EP violation to produce a new result. The expected sensitivity of the differential red shifts as measured by the three clocks that are within six solar radii is at the level of 10^{-13} of the EP, or about six orders of magnitude larger than the GP-A experiment. Thus results of SpaceTime will improve the current state of art in EP violation by an order of magnitude, as well as improving on the results of Webb *et al.* by four orders of magnitude, beyond the capability of all existing and future earth-bound clock experiments.

To improve the measurement sensitivity, our instrument consists of three clocks based on three different atomic species that can be inter-compared for individual signatures. To reduce the influence of systematic errors that can mimic our signal, the three clocks share the same environment. To improve the source of the signal, the tri-clock instrument flies to within six solar radii of the largest body of matter in the solar system, the Sun. Thus the entire experiment is designed to provide a clean and unambiguous result, based on a technology that is proven, and has an outstanding chance for success. Finally, the spinning spacecraft, moving at 300 km/s, or 1/1000 of the speed of light, at its closest approach will test another important question with fundamental underpinning: Is Lorentz symmetry robust, or does it fail at some limit? This question is important since string theory, and theories that extend beyond the Standard Model [17], result in physics without Lorentz and other global symmetries such as CPT.

Beyond this, as mentioned above, a consequence of a changing α is that either c, the speed of light, or e, the charge of the electron, or h, Planck's constant, must change. Theories based on a changing velocity of light have received considerable attention since they solve the outstanding problems in cosmology: the horizon, flatness, cosmological constant, entropy, and homogeneity problems [5]. They nonetheless violate Lorentz invariance. SpaceTime will provide a ten-fold sensitivity for a test of Lorentz invariance, as compared to an earth bound test, due to the order of magnitude smaller orbital speed of earth [18].

2 The Instrument

In the strongly time-dilated spacetime curvature at six solar radii (4.2 Gm), time runs slower than on Earth by about one half microsecond per second. Three atomic clocks based on hyperfine transitions of Hg^+ ($Z = 80$), Cd^+ ($Z = 48$), and Yb^+ ($Z = 70$) are different in their electromagnetic composition (given by the Casimir factor) and will be simultaneously monitored during a solar flyby to determine whether these different clocks will measure the same time interval near the Sun. The atomic clock hardware for the SpaceTime mission is a modification of the linear ion trap frequency standard (LITS) currently being deployed in the Deep Space Network stations worldwide. A laboratory prototype has shown ultra-stable operation in a package far smaller than other clock technologies and represents the state of the art for atomic clocks.

Atomic clocks based on hyperfine transitions and ion traps are the most suitable technology for space applications. This is because of the inherent simplicity of this approach, which does not rely on resonant cavities. In lamp based trapped ion clocks, as in the SpaceTime instrument, the risk associated with the use of lasers in space is eliminated. Ions confined in electromagnetic traps are significantly shielded from environmental perturbations such as collisions with the walls or each other. The relatively large hyperfine splitting of singly ionized systems also reduce their sensitivity to ambient magnetic fields, as compared with atoms with smaller hyperfine frequencies.

The classical ion trap consisting of a three-electrode structure made with hyperbolic electrodes confines charged particles of particular charge to mass

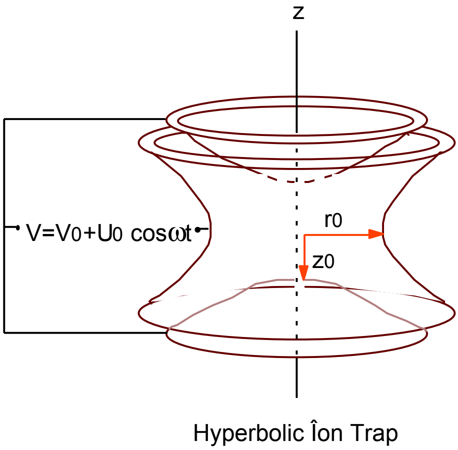

Fig. 1. Ion Trap

ratios based on the applied dc and rf potentials (see Fig. 1). In this geometry, ions are confined in a spherical region as a result of the applied ponderomotive forces.

A geometry based on linear electrodes, first introduced at JPL for clock applications, improves the clock stability by providing a geometry whereby the temperature (kinetic energy) of the ions resulting from the micro-motion in the trap is reduced [19]. This configuration was further refined at JPL [20] to put the ability to move the charged particles from one region of space to another, to separate the ion preparation region from the region where the microwave field produced by a local oscillator (LO) interacts with the clock transition of ions (Fig. 2). By separating these regions it is possible to significantly reduce the requirement of magnetic shielding which must protect the ions undergoing interaction with the microwave field. Higher pole traps are also employed in order to further reduce ion density space-charge related ion-heating. This is key to the reduction of the size and weight of the clock, parameters that are particularly important for space instruments.

The instrument for this mission is composed of three ion trap clocks in a package where much of the hardware is common to all of the clocks. Because some of the clock systematic frequency perturbations will be common to all three clocks and will have a characteristic signature that can be identified and removed from the difference of the clock frequencies, relative stabilities to 10^{-16} in the inter-comparison can be reached. The local oscillator (LO) will simultaneously interrogate each of the three clock transitions thereby removing LO noise in the inter-comparison, and greatly improving short-term clock noise so that 10^{-16} resolution in the difference in clock rates can be obtained within the 15-hour close encounter. Because ion-trap-based clocks are relatively immune to temperature and magnetic field changes, a simple, robust electronics package is sufficient for ultra-stable operation.

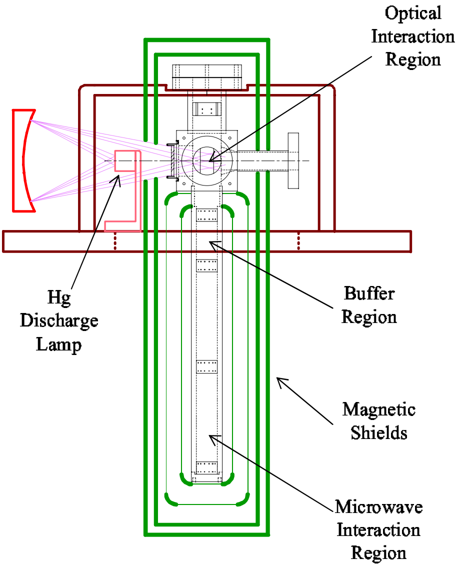

Fig. 2. Schematic of Linear Ion Trap

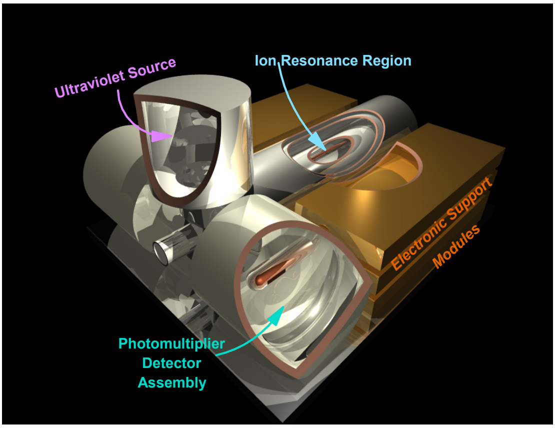

Fig. 3. Tri-clock Instrument

The basic architecture of the "tri-clock" instrument is three LITE (Linear Ion Trap Extended) units, each operating with a single element Hg^+, Cd^+ or Yb^+, and will be packaged into one housing with many shared components for mass reduction. This configuration is shown in Fig. 3. Each separate clock is based upon a linear multi-pole trap [20]. For optical state-selection, ions are trapped around the rf quadrupole electric field node along the centerline where they are prevented from escaping by dc fields applied at each end. By applying dc positive bias to all trap rods in one region along the length of the trap, ions can be excluded from that region and 'transported' into another section where the rods are at dc ground. Ions can thus be moved from one end of the trap to the other. This allows the optical state selection and interrogation to be carried out in an unshielded region while the much more critical clock hyperfine resonance is probed in a small, well shielded region, away from magnetic optical components and openings in the shields for light entry and exit. The ion-number (space-charge) induced frequency pulling is reduced by more than a factor of 20 in the multi-pole arrangement as compared to the linear quadrupole [21,20,22,23].

The three traps will be operated with a common rf voltage source so that related trapping forces confine the three different ion species. In this way small variations in the trapping strength will affect each ion cloud in a characteristic manner that can be readily identified. Another unique feature of this clock comparison is the use of the ultra-stable local oscillator. Space-qualified quartz oscillators achieve short-term stabilities of 10^{-13} over tens of seconds averaging intervals. This will limit a conventional high performance atomic clock to about 10^{-13} at 1 second averaging time, falling from there as $\tau^{-1/2}$ where τ is the averaging interval in seconds. For the clock comparison at the near-solar flyby, the largest change in gravitational potential occurs over a 15-hour period, i.e., 54 000 s. This LO-limited performance gives 4×10^{-16} at 15 hours and falls short of the design goal. We have demonstrated atomic clock performance at $(2-3) \times 10^{-14}$ at one second but LO noise degrades the performance for a single operating atomic clock. For a comparison between two or more clocks, however, a single LO can be used to interrogate all clock transitions simultaneously, and the LO noise will be common. This common noise in individual atomic line-center measurements will not be present in the differences of these and we can recover the $(2-3) \times 10^{-14}/\sqrt{\tau}$ and reach the 10^{-16} stability level in 15 hours averaging.

The tri-clock measurement offers a suppression of other common mode frequency shifts of the three atomic transitions. The suppression of systematic frequency pulling can also be applied to variations of the solar magnetic field along the spacecraft trajectory. This approach will save mass and power in magnetic shielding. A set of four layers of magnetic shields will enclose the clock resonance tube. An additional layer will house the final package. Since the unshielded Hg^+ atom sensitivity is about 2×10^{-13}/mG (at an operating point of 50 mG), 20×10^{-13}/mG for Yb^+, and 15×10^{-13}/mG for Cd^+, a shielding factor of 10^7 is required to reduce a 1-G solar field variation during the spacecraft flyby to below one part in 10^{16} relative clock stability. A 1-G field variation might be expected during the solar flyby. This level of shielding is very difficult to achieve within the mass and power budget.

The differential response of the three clocks to a common field variation has a characteristic signature that will identify this systematic shift and will enable its removal in post analysis. The magnetic sensitivity of the three hyperfine levels is well understood in the atomic physics of the clock transitions. The change of the clock frequency as the operating field changes by δH_0 is given by $\delta y \equiv \delta f/f_0 = 2\beta H_0 \delta H_0$ where the constant β describes the field sensitivity of each of the three clock transitions. The atoms with a smaller hyperfine splitting f_0 shift more. Note that this behavior is very different from the sensitivity to a change in α as given in [12]. In that paper it is shown that the atoms with larger atomic number Z shift more with a change in α than the low Z atoms.

The two simultaneous equations for the variation of the difference frequencies are

$$\delta y_{AB} = \Big(L(Z_A) - L(Z_B)\Big)\frac{\delta\alpha}{\alpha} + \left(1 - \frac{\beta_B f_A}{\beta_A f_B}\right)\frac{2\beta_B}{f_A}\frac{H_0 \delta H}{S} ,$$
$$\delta y_{AC} = \Big(L(Z_A) - L(Z_C)\Big)\frac{\delta\alpha}{\alpha} + \left(1 - \frac{\beta_C f_A}{\beta_A f_C}\right)\frac{2\beta_B}{f_A}\frac{H_0 \delta H}{S} . \quad (3)$$

We have taken the variation of the clock transitions with operating field, H_0, to be given by $f = f_0 + \beta H_0^2$ and the shielding factor for external fields to be S, i.e., $\delta H_0 = \delta H/S$. δH is the variation of the solar magnetic field along the spacecraft trajectory. The α sensitivities, $L(Z)$, are found in Fig. 1 of [12].

For the hyperfine clock transitions in Hg, Cd, and Yb, these equations can be inverted to solve for $\delta\alpha/\alpha$ and $(2\beta_A/\nu_A)H_0\delta H/S$ along the trajectory of the near-Sun flyby. Thus, even with imperfect magnetic shielding and the accompanying clock frequency pulling, an unambiguous variation of α could be extracted.

2.1 Temperature Induced Frequency Shifts

Ambient temperature changes of the clocks can cause spurious frequency pulling δy_{AB} and δy_{AC} and must be completely removed to the 10^{-16} level. Unlike magnetic sensitivities, which can be to a large extent understood as incomplete shielding of the atomic transition, temperature-induced frequency shifts are more difficult to predict from first principles. The only definitive measurement of temperature sensitivity must be carried out with a fully assembled and operating system. The differential sensitivity coefficients to be used in separating any observed effect from a temperature induced δy_{AB} and δy_{AC}, must be generated in-situ. Once these sensitivities are measured, we can use the two return data channels to distinguish temperature effects from any observed violations.

Some temperature effects have very clear signatures, completely distinguishable from any a variation along the spacecraft trajectory. For example, ion temperature variations will lead to clock frequency changes via second-order Doppler shifts, by an amount proportional to $-kT/mc^2$ where T is the ion temperature and m is the ion mass. Any temperature change, δT, common to all three ionic species will shift the three clock frequencies by an amount inversely proportional to their mass. This will allow this systematic frequency offset to be removed as in the magnetic case above. For these shifts,

$$\delta y_{AB} = \Big(L(Z_A) - L(Z_B)\Big)\frac{\delta\alpha}{\alpha} + \left(1 - \frac{m_A}{m_B}\right)\frac{k\delta T}{m_A c^2} ,$$
$$\delta y_{AC} = \Big(L(Z_A) - L(Z_C)\Big)\frac{\delta\alpha}{\alpha} + \left(1 - \frac{m_A}{m_C}\right)\frac{k\delta T}{m_A c^2} , \quad (4)$$

showing that these temperature variations can be separated from the variations that come from a non-zero $\delta\alpha/\alpha$ along the solar flyby trajectory. We have assumed no mass dependent heating, δT, which will almost certainly be present.

However, a pre-launch ground measurement will be carried out to catalog differential frequency shifts vs rf trap level, buffer gas pressure, etc.

2.2 Mission Design

The only economical technique to get sufficient change in velocity to fly near the Sun is to go via Jupiter. This is because the angular momentum associated with the orbiting earth must be lost so the spacecraft will fall to the Sun in a reasonable length of time. Thus, SpaceTime will launch in a direct transfer orbit to Jupiter and then a fast trajectory to the Sun. A kick stage is integrated with the spacecraft on a "spin table" that spins the entire integrated package during the launch. The spinning spacecraft does not have to be despun following injection, as with a typical three-axis stabilized spacecraft. This eliminates the mass and reliability penalties of a despin hardware.

Figure 4 illustrates the entire interplanetary trajectory to the Sun including the first leg after injection. The time tics are 50-day intervals. Approaching Jupiter, a precision orbit determination is completed using only radio tracking data, and a precise final aiming maneuver is completed. The gravity assist flyby is used to: 1) reduce (almost canceling) the trajectory angular momentum, allowing the spacecraft to fall into a 6-R_S perihelion, 2) rotate the plane of the heliocentric orbit to a final inclination of 90.0 degrees and 3) establish the time of perihelion to produce a quadrature trajectory geometry (Sun-spacecraft-Earth angle = 90.0 degrees) at perihelion. This latter condition is fundamental to the spacecraft architecture, which always has the shield pointed at the Sun and the high gain antenna (HGA) pointed at Earth.

Following the Jupiter flyby, the spacecraft is on its final trajectory toward the perihelion. The perihelion flyby trajectory is shown in Fig. 4 from $P-24$ to $P+24$ hr. This is the prime data acquisition period for the mission. The view in Fig. 4 is from Earth illustrating the effects of the quadrature trajectory geometry by the schematic drawings of the spacecraft. The spacecraft is a spinning drum with the direction of its spin axis toward the Earth (out of the page). The thermal shield for the spacecraft, as the spacecraft spins, maintains its orientation toward the Sun at all times protecting the sensitive elements from the extreme thermal

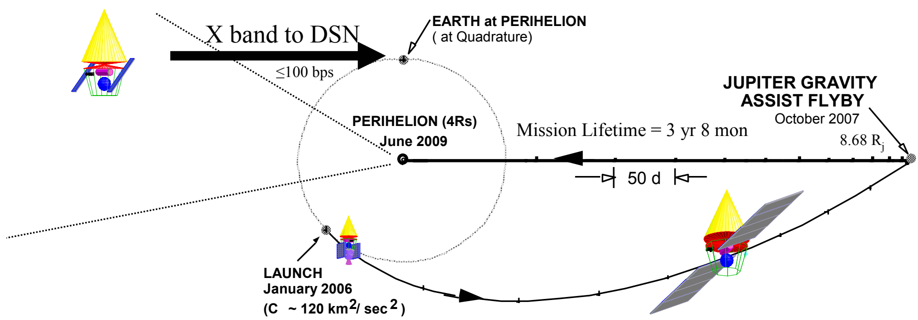

Fig. 4. Spacecraft trajectory

environment. This is a passive attitude control technique that simplifies the control of the spacecraft and allows a very robust design in this otherwise hostile environment.

It is interesting to point out that the most challenging aspect of the mission, affecting orbital trajectory and the number of passes (single) by the Sun is the power requirements. Because of the extreme heat encountered near the Sun, solar panels, even those designed for high temperature, cannot be used. Instead, a bank of batteries must provide the needed power to the spacecraft systems, and the instrument. The mass associated with the batteries ultimately limits the choices of a trajectory with a given launch vehicle, as well as the size of the spacecraft and associated systems. This ironic limitation (shortage of power while so near the Sun) is the major design issue that affects virtually all aspects of the mission.

3 Conclusion

We have briefly discussed a mission design study based on the inter-comparison of the oscillation frequencies of three atomic clocks based on three different species of singly ionized atoms. By flying this instrument to within six solar radii of the Sun it is possible to search for a variation of fine structure constant to a level that is not accessible to earth-based instruments. At this point two other points regarding this approach are worth noting. First, one may ask the question why the choice of atomic clocks, as opposed to other instruments (see a description of LATOR mission in this volume [24]). As briefly mentioned above, and discussed elsewhere in this volume (see, for example, the paper by Flambaum *et al.*) the detail of theories that predict a temporal or spatial variation in fine structure constant, such as M-theory or theories based on varying c or e, are rather tentative. Experimental tests of these theories based on a search for varying α then must produce direct and unambiguous results to be most valuable. The three-clock comparison discussed here is indeed such an approach. As discussed above, each atomic clock will drift in a specific manner with varying α and inter-comparison of these variations assures that an observed signal produces a clear result. Secondly, the technology of atomic clocks is well developed, and a space test based on clocks has an inherently large probability of success.

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References

1. J.K. Webb et al., Phys. Rev. Lett. **87**, 091301 (2001).
2. T. Damour, A. M. Polyakov, Nucl. Phys. B. **423**, 532 (1994).
3. P.A.M. Dirac, Nature **139**, 323 (1937).
4. J.D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
5. J.D. Barrow, J. Magueijo, astro-ph/9811072.
6. H. Sandvik, J.D. Barrow, J. Magueijo, astro-ph/0107512.
7. J.W. Moffat, astro-ph/01093501.
8. D. Youm, hep-th/0108237
9. J.D. Bekenstein, Phys. Rev. D **66**, 123514 (2002).
10. T. Damour, Astro. Phys. Space Sci. **283**, 445 (2003).
11. R. Vessot et al., Phys. Rev. Lett. **45**, 2081 (1980).
12. J.D. Prestage, R.L. Tjoelker, L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
13. H. Marion et al. Phys. Rev. Lett. **90**, 150801 (2003)
14. S. Bize et al. Phys. Rev. Lett. **90**, 150802 (2003)
15. T. Damour, F. Dyson, Nucl. Phys. B. **480**, 37 (1996).
16. E.G. Adelberger, Class. Quant. Grav. **18**, 2397 (2001).
17. V.A. Kostelecky, *CPT and Lorentz Symmetry*. World Scientific, Singapore, 1999.
18. R. Bluhm, V.A. Kostelecky, C. Lane, N. Russell, Phys. Rev. Lett. **88**, 090801 (2002)
19. J.D. Prestage, G. J. Dick and L. Maleki, J. Appl. Phys. **66**, 1013 (1989).
20. J.D. Prestage, R.L. Tjoelker and L. Maleki, Topics Appl. Phys. **79**, 195 (2001), Frequency Measurement and Control, ed. A. N. Luiten (Springer, Heidelberg, 2001).
21. J.D. Prestage, R.L. Tjoelker, L. Maleki, Proc. 2000 IEEE Freq. Control Symp., p. 459 (2000).
22. J.D. Prestage, S. Chung, E. Burt, L. Maleki, and R.L. Tjoelker, Proc. 2002 IEEE Freq. Control Symp., p. 706 (2000).
23. R.L. Tjoelker, J.D. Prestage, P. K. Koppang, and T. B. Swanson, Proc. 2003 IEEE Freq. Control Symp., p. 1066 (2003).
24. S.G. Turyshev et al., 35 Years of Testing Relativistic Gravity: Where Do We Go from Here?, Lect. Notes Phys. **648**, 311–330 (2004)